Virtual Reference Feedback Tuning for Non Minimum Phase Plants

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Abstract

Model Reference control design methods fail when the plant has one or more non minimum phase zeros that are not included in the reference model, leading possibly to an unstable closed loop. This is a very serious problem for data-based control design methods, where the plant is typically unknown. In this paper, we extend the Virtual Reference Feedback Tuning method to non minimum phase plants. This extension is based on the idea proposed in [12] for Iterative Feedback Tuning. We present a simple two-step procedure that can cope with the situation where the unknown plant may or may not have non minimum phase zeros.

Key words: Model reference control; data-based control; VRFT; non-minimum phase systems; flexible reference model.

1 Introduction

In the past two decades, a number of data-based control design methods have been proposed [8,7,5,10], where a parametrized controller structure is chosen a priori, and the controller tuning is based directly on input and output data collected on the plant without the use of a model of this plant. These data-based controller tuning methods may fail when the plant has Non Minimum Phase (NMP) zeros that are not included in the *desired reference model*, leading possibly to an unstable closed loop. The only safe way to avoid it is by including the NMP zeros in the desired reference model. Doing this a priori, when choosing the reference model, requires the knowledge of this zero with good precision, which in turn may be a rash hypothesis even for model-based design.

In the context of Virtual Reference Feedback Tuning (VRFT) [5] the authors of [17] proposed the identification of the process to find the possible NMP zeros. If there are NMP zeros they are included in the reference model and then the Virtual Reference Feedback Tuning is applied. The objective of this paper is to present an alternative way to obtain the NMP zeros of the process within the framework of data-based control design, avoiding the identification of the process poles and stable zeros. We propose the use of a *flexible reference model*, inspired by a similar solution proposed for Iterative Feedback Tuning (IFT) [7] to overcome the problems of NMP zeros [12]. The *flexible reference model* has the same poles as the desired reference model, but the parameters of its numerator polynomial are free. The main difference with [17] is that we estimate fewer parameters, thus enabling higher precision.

We propose a two-step procedure to deal with the situation where the unknown plant may contain NMP zeros. In the first step a VRFT criterion is minimized simultaneously with respect to the parameters of a flexible reference model with free numerator coefficients and with respect to the controller parameters. Under certain assumptions, we show that if the true unknown plant has NMP zeros, then the estimated numerator coefficients of the flexible reference model converge to these NMP zeros. If this first step converges to a numerator polynomial of the reference model that contains NMP zeros, then the desired reference model is modified so as to include these NMP zeros; the second step then consists of minimizing the standard VRFT criterion with this modified reference model. If the first step converges to a numerator polynomial of the reference model that does not contain NMP zeros, then the second step consists of minimizing the standard VRFT criterion with the initial desired reference model.

The paper is organized as follows. Definitions and the problem formulation are presented in Section 2. Section 3 reviews the standard VRFT method and the proposed flexible criterion for VRFT is then presented in Section 4, while Section 5 shows some examples of the application of the proposed method. In the end, we present some conclusions.

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2 Preliminaries

Consider a linear time-invariant discrete-time single-input-single-output process

$$y(t) = G_0(z)u(t) + v(t), \tag{1}$$

where z is the forward-shift operator, $G_0(z)$ is the process transfer function, $u(t)$ is the control input and $v(t)$ is a quasi-stationary noise process which can be written as $v(t) = H_0(z)e(t)$ where $e(t)$ is white noise with variance σ_e^2. Both transfer functions, $G_0(z)$ and $H_0(z)$, are rational and causal and it is assumed that $G_0(z)$ has a nonzero static gain.

This process is controlled by a linear time-invariant controller which belongs to a given - user specified - controller class $\mathscr{C}$ that is linearly parametrized: $\mathscr{C} = \{C(z,\rho) = \rho^T\beta(z), \rho \in \mathbb{R}^n\}$, where $\beta(z)$ is a n-column vector of fixed causal rational functions, whose poles are strictly inside the unit circle except for possible poles at $z = 1$. This class is such that $C(z)G_0(z)$ has positive relative degree for all $C(z) \in \mathscr{C}$; equivalently, the closed loop is not delay-free. The control action $u(t)$ can be written as

$$u(t) = C(z,\rho)(r(t) - y(t)), \tag{2}$$

where $r(t)$ is a reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise, that is $\bar{E}[r(t)e(s)] = 0 \ \forall t,s$. Here $\bar{E}[\cdot]$ denotes $\bar{E}[f(t)] \triangleq \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E[f(t)]$ with $E[\cdot]$ denoting expectation [13]. The system (1)-(2) in closed loop becomes

$$y(t,\rho) = T(z,\rho)r(t) + S(z,\rho)v(t)$$
$$T(z,\rho) = \frac{C(z,\rho)G_0(z)}{1 + C(z,\rho)G_0(z)} = C(z,\rho)G_0(z)S(z,\rho)$$

where we have now made the dependence on the controller parameter vector ρ explicit in the output signal $y(t,\rho)$.

Model reference control design consists of specifying a "desired" closed loop transfer function $\bar{M}(z)$, which is known as the *reference model*, and then solving the following optimization problem

$$\min_{\rho} J^{MR}(\rho) \tag{3}$$
$$J^{MR}(\rho) \triangleq \bar{E}\left[((T(z,\rho) - \bar{M}(z))r(t))^2\right]. \tag{4}$$

The model matching controller $C_d^{MR}(z)$ is the one that allows the closed loop system to match exactly $\bar{M}(z)$:

$$C_d^{MR}(z) = \frac{\bar{M}(z)}{G_0(z)(1 - \bar{M}(z))}. \tag{5}$$

Should the model matching controller $C_d^{MR}(z)$ be put in the control loop, the objective function would evaluate to zero. However, this model matching controller may not be causal, or may produce an internally unstable closed loop system through the cancellation of NMP zeros, both of which would be disastrous. Thus, the choice of the reference model $\bar{M}(z)$ must be made under some constraints to prevent these disasters; these constraints are directly obtained from (5). In order for $C_d^{MR}(z)$ to be causal, the relative degree of $\bar{M}(z)$ cannot be smaller than that of the plant $G_0(z)$. To prevent the unstable pole-zero cancellation, the reference model $\bar{M}(z)$ must have the same unstable zeros as the plant. So, the choice of the reference model requires a priori knowledge of an overbound for the relative degree of the plant and knowledge of the location of its unstable zeros, if any.

Data-based control methods and direct adaptive control methods address the minimization of the criterion (4) directly from data collected from the system, without deriving a process model from this data [7,9,5,10]. It is then not always possible to assume a priori knowledge of the existence of NMP zeros, and certainly not their positions in case they do exist. The choice of an appropriate reference model is then compromised. Thus most data-based design methods tend to fail when applied to non minimum phase plants. In this paper we propose a solution to this problem, extending the VRFT [5] method to cope with NMP plants. We start by presenting the standard VRFT method.

3 The standard VRFT method

The Virtual Reference Feedback Tuning Method (VRFT) is a direct (non iterative) method that consists of minimizing a criterion whose minimum is the same as that of $J^{MR}(\rho)$ under certain ideal conditions. This new criterion is quadratic, and thus easy to minimize whereas other data-based methods require iterative procedures and may suffer from convergence to local minima [1]. The method has been applied to different problems, including nonlinear and multivariable systems [3,15]. An application to a real system is presented in [4] and some thoughts regarding its application to unstable and noisy processes are presented in [16].

3.1 The ideal case

The Virtual Reference Feedback Tuning method was initially proposed for an ideal case where the collected signals are not affected by noise and the model matching controller (5) belongs to the considered controller set, that is:

Assumption 1 $C_d^{MR}(z) \in \mathscr{C}$ *or, equivalently,*

$$\exists \rho_d : C(z,\rho_d) = \rho_d^T\beta(z) = C_d^{MR}(z)$$

The VRFT method can be described as follows. Through either an open loop or a closed loop experiment, input data $u(t)$ and output data $y(t)$ are collected on the actual process. Given the measured $y(t)$, we define the *virtual reference* signal $\bar{r}(t)$ such that $\bar{M}(z)\bar{r}(t) = y(t)$. This means that, if the system were in closed loop with the model matching controller $C_d^{MR}(z)$, and $\bar{r}(t)$ were applied to the reference, the output data would be the same as the data $y(t)$ that have been collected in the experiment. Should the data have been collected like this, the reference tracking error would have been given by $\bar{e}(t) = \bar{r}(t) - y(t)$. This $\bar{e}(t)$ is the signal that would have fed the model matching controller in this virtual experiment. We thus have input and output data ($\bar{e}(t)$ and $u(t)$ respectively) of the model matching controller $C_d^{MR}(z)$ and we can use these data to identify it. The identification is performed by minimizing the following criterion

$$
\begin{aligned}
J^{VR}(\rho) &= \bar{E}\left[u(t) - C(z,\rho)\bar{e}(t)\right]^2 \\
&= \bar{E}\left[u(t) - \left(C(z,\rho)\frac{1-\bar{M}(z)}{\bar{M}(z)}\right)y(t)\right]^2
\end{aligned} \tag{6}
$$

Since $C(z,\rho)$ is linear in ρ, the criterion in (6) is a quadratic function of the parameter vector ρ and hence the solution of the optimization problem can be obtained through the application of least squares, that is, by the following calculation:

$$
\hat{\rho} = \bar{E}\left[\varphi(t)\varphi(t)^T\right]^{-1}\bar{E}\left[\varphi(t)u(t)\right] \tag{7}
$$

where $\varphi(t) = \beta(z)\bar{e}(t)$. This is the key advantage of the VRFT criterion (6) over the MR criterion (4), and hence of VRFT over other data-based methods, like IFT or CbT, which are iterative.

Under Assumption 1 the parameter value ρ_d is the global minimum of both criteria, (4) and (6), since both evaluate to zero at $\rho = \rho_d$. It is also easy to demonstrate that this global minimum is unique, for both criteria, provided that the corresponding regression vector is persistently exciting [1,5]. Therefore, under quite reasonable assumptions the VRFT method can identify the model matching controller (5) exactly.

3.2 *The non ideal case*

Some techniques have been created to minimize the effects of the noise on the collected data and the impossibility of achieving the model matching controller. When Assumption 1 does not hold, the minima of the two criteria (4) and (6) are not the same, but they can be made close to one another by proper filtering of the signals $u(t)$ and $\bar{e}(t)$, as shown in [5]. The appropriate filter $L(z)$ is defined by:

$$
|L(z)|^2 = |1-\bar{M}(z)|^2|\bar{M}(z)|^2\frac{\Phi_r}{\Phi_u}, \tag{8}
$$

where Φ_u is the power spectrum of the signal $u(t)$ and Φ_r is the power spectrum of $r(t)$. In this case, the parameter vector ρ is estimated by

$$
\hat{\rho} = \bar{E}\left[\varphi_L(t)\varphi_L(t)^T\right]^{-1}\bar{E}\left[\varphi_L(t)u_L(t)\right]
$$

where $\varphi_L(t) = L(z)\varphi(t)$ and $u_L(t) = L(z)u(t)$.

In the presence of noise, an instrumental variable can be used instead of the standard least squares solution, in which case equation (7) is replaced by $\hat{\rho} = \bar{E}\left[\zeta(t)\varphi_L(t)^T\right]^{-1}\bar{E}\left[\zeta(t)u_L(t)\right]$ where $\zeta(t)$ is a n-vector of instrumental variables; see [5] for details.

3.3 *The NMP zeros problem*

The VRFT method can be seen as a method that searches the parameters of a fixed structure controller to make the behavior of the closed loop system as close as possible to the closed loop system with $C_d^{MR}(z)$. When the process has NMP zeros and the reference model does not, the model matching controller $C_d^{MR}(z)$ is unstable; therefore the closed loop system with $C_d^{MR}(z)$ is internally unstable. So, the parameters of the VRFT controller will tend to values that make the closed loop system unstable. This can occur even with a controller structure with fixed stable poles because the source of the problem is that the model matching controller is unstable, as illustrated in the example in Section 5.1.1.

When the reference model does not possess the NMP zeros of the plant, Assumption 1 is not satisfied, and then we should use the filter $L(z)$ of equation (8). However, the idea of using this filter is based on the approximation $C(z,\rho) \approx C_d^{MR}(z)$, which makes no sense in this situation. To avoid this problem, the NMP zeros of the plant must be included in the reference model, thereby yielding a stable model matching controller and a stable closed loop system. In the next section we present a modification of the optimization criterion of VRFT in order to successfully cope with possible NMP zeros of the plant. A data-based framework will be used to obtain the NMP zeros of the plant so that they can be included in the reference model.

4 Flexible criterion for VRFT

In this section we propose the use of a reference model with free parameters in the numerator to be used with the VRFT method, based on the solution proposed in [12] for the IFT method. The parametrized class of reference models is described by

$$
\mathscr{M} = \{M(z,\eta) = \eta^T F(z)\}, \tag{9}
$$

where $\eta \in \mathbb{R}^q$ is a vector of free parameters and $F(z)$ is a q-vector of rational functions. By replacing the fixed reference model $\bar{M}(z)$ by $M(z,\eta)$ and by applying the filter $L(z)$, the VRFT criterion (6) is changed into [2]

$$
J_0^{VR}(\eta,\rho) = \bar{E}\left\{L(z)\left[u(t) - \left(\frac{1-M(z,\eta)}{M(z,\eta)}C(z,\rho)\right)y(t)\right]\right\}^2. \tag{10}
$$

In this formulation the denominator of the reference model is assigned, while the numerator is left free. We shall see that the optimization of $J_0^{VR}(\eta,\rho)$ can then "find" the zeros of the plant, and particularly the NMP zeros. If the number of free parameters q equals the order of the numerator of $M(z,\eta)$, the numerator is entirely free and the formulation becomes conceptually equivalent to a pole assignment design.

In the standard VRFT method, the model matching hypothesis - Assumption 1 - is crucial. Our analysis for this new design criterion requires a similar assumption. Assumption 2 below states that there exists, within the class of reference models $\mathscr{M}$ considered, one reference model for which model matching is possible.

Assumption 2 *There exists a pair* (η^*,ρ^*) *such that* $J_0^{VR}(\eta^*,\rho^*)=0$*, or, equivalently,*

$$\exists\eta^*,\rho^*:\ C(z,\rho^*)=\frac{M(z,\eta^*)}{[1-M(z,\eta^*)]G_0(z)}. \qquad (11)$$

The next theorem shows that the reference model $M(z,\eta^*)$ that satisfies Assumption 2 contains all NMP zeros of $G_0(z)$.

Theorem 1 *Let $B(z)$ be the least common denominator of the elements of $\beta(z)$. Let Assumption 2 be satisfied. Then the NMP zeros of $G_0(z)$ are also zeros of $M(z,\eta^*)$.*

Proof Let $G_0(z)=\frac{n_G(z)}{d_G(z)}$ be a coprime factorization of $G_0(z)$, where $n_G(z)$ and $d_G(z)$ are polynomials. From (11) we have that

$$C(z,\rho^*)=\frac{n_M(z,\eta^*)d_G(z)}{[d_M(z)-n_M(z,\eta^*)]n_G(z)}, \qquad (12)$$

where $M(z,\eta^*)=\frac{n_M(z,\eta^*)}{d_M(z)}$. Since $G_0(z)$ has a nonzero steady state gain, $n_G(z)$ has no zero at $z=1$. Since the poles of $C(z,\rho^*)$ (i.e. the roots of $B(z)$) are either at $z=1$ or strictly inside the unit circle, $B(z)$ and $n_G(z)$ have no common unstable roots. Therefore, since the left hand side of (12) is stable, any unstable root of $n_G(z)$ must be canceled by a root of $n_M(z,\eta^*)$. □

Under Assumption 2, it follows that

$$\arg\min_{\eta,\rho} J_0^{VR}(\eta,\rho)=\arg\min_{\substack{\eta,\rho\\(\eta,\rho)\neq\{0,0\}}}\tilde{J}_0^{VR}(\eta,\rho)=(\eta^*,\rho^*) \qquad (13)$$

where

$$\tilde{J}_0^{VR}(\eta,\rho)=\bar{E}\left[L(z)M(z,\eta)u(t)-L(z)C(z,\rho)(1-M(z,\eta))y(t)\right]^2. \qquad (14)$$

Given the linear parametrization of both the controller and the reference model, $\tilde{J}_0^{VR}(0,0)=0$. Thus, the multiplication by $M(z,\eta)$ has created an additional - and undesired - global minimum at the origin. This is why the right hand side of (13) is subjected to a constraint that excludes this undesired minimum $(\eta,\rho)=\{0,0\}$. In most control applications, a natural constraint exists which automatically does that: the reference model must have steady-state gain $M(\eta,1)=1$. Note that, if $M(z,\eta)=\frac{\eta_1 z^q+\eta_2 z^{q-1}+\ldots\eta_q}{z^m+a_1 z^{m-1}+\ldots+a_m}$, then $M(1,\eta)=\frac{\eta_1+\eta_2+\ldots\eta_q}{1+a_1+\ldots+a_m}=1$, which needs at least one coefficient of the η-vector to be nonzero, hence excluding the undesired minimum.

It now follows from Theorem 1 that, under Assumption 2, the minimization (13) of $\tilde{J}_0^{VR}(\eta,\rho)$ yields a minimum (η^*,ρ^*) such that $M(z,\eta^*)$ contains all NMP zeros of $G_0(z)$. We have thus produced a data-based optimization procedure that detects the NMP zeros of the plant without utilizing a full order model identification procedure, the only assumption being that the controller structure is such that the desired closed loop poles can be achieved. On the other hand, by focusing on the identification of the NMP zeros only, better precision in their location can be expected [6,14].

4.1 Implementation issues

Inserting (9) and $C(z,\rho)=\rho^T\beta(z)$ into (14) yields

$$\tilde{J}_0^{VR}(\eta,\rho)=E\{\eta^T F(z)[u_L(t)+\rho^T\beta(z)y_L(t)] -\rho^T\beta(z)y_L(t)\}^2 \qquad (15)$$

Since the argument in (15) is bilinear in η and ρ, the minimization of $\tilde{J}_0^{VR}(\eta,\rho)$ can be treated as a sequence of least squares problems [13]:

$$\hat{\eta}^{(i)}=\arg\min_{\eta}\tilde{J}_0^{VR}(\eta,\hat{\rho}^{(i-1)}) \qquad (16)$$

$$\hat{\rho}^{(i)}=\arg\min_{\rho}\tilde{J}_0^{VR}(\hat{\eta}^{(i)},\rho) \qquad (17)$$

where each least squares step has an explicit solution:

$$\hat{\eta}^{(i)}=\bar{E}\left\{[F(z)w(\hat{\rho}^{(i-1)},t)][F(z)w(\hat{\rho}^{(i-1)},t)]^T\right\}^{-1}\times \bar{E}\left\{[F(z)w(\hat{\rho}^{(i-1)},t)][C(z,\hat{\rho}^{(i-1)})L(z)y(t)]\right\}, \qquad (18)$$

$$\hat{\rho}^{(i)}=\bar{E}\left\{[\beta(z)v(\hat{\eta}^{(i)},t)][\beta(z)v(\hat{\eta}^{(i)},t)]^T\right\}^{-1}\times \bar{E}\left\{[\beta(z)v(\hat{\eta}^{(i)},t)][M(z,\hat{\eta}^{(i)})L(z)u(t)]\right\}, \qquad (19)$$

$$w(\rho,t)\triangleq L(z)[u(t)+\rho^T\beta(z)y(t)],$$
$$v(\eta,t)\triangleq L(z)[1-\eta^T F(z)]y(t)$$

This sequential least squares algorithm is guaranteed to converge at least to a local minimum [13,18].

Theorem 2 *The algorithm (16)-(17) converges to an extremum of $\tilde{J}_0^{VR}(\eta,\rho)$.*

Proof It is clear from the algorithm that $\tilde{J}_0^{VR}(\eta,\rho)$ is a strictly decreasing function of the sequence $\hat{\eta}^{(i)}$, $\hat{\rho}^{(i)}$; it can thus be taken as a Lyapunov function. The convergence is then a standard result from Lyapunov theory [11]. □

When the data are collected in closed loop, it is natural to use the parameters of the controller that is operating in the loop during the experiment as the initial value of the sequential least-squares algorithm. But this is not the only possible choice. The algorithm also needs an initial value for the filter $L(z)$, which depends on $M(z,\eta)$. One possible choice is to use $\bar{M}(z)$ for this purpose. It is also worth stressing that even though the minimization algorithm is iterative, the data from the system are collected just once, thereby keeping the "one-shot" property of the VRFT method. Notice that if $M(z,\hat{\eta})$ contains NMP zeros, then the standard VRFT used with the modified $\bar{M}_m(z)$ will yield an unstable filter $\bar{M}_m^{-1}(z)$, when searching for $\bar{r}(t)$. This issue can be dealt with by multiplying $L(z)$ with an all-pass frequency weighting filter, which leaves the objective function $J^{VR}(\rho)$ (6) unchanged; the signals needed by the VRFT method are then obtained from stable filters [7,17].

4.2 Two-step procedure

We have shown that the global minimum of the flexible criterion $\tilde{J}_0^{VR}(\eta,\rho)$ corresponds to a reference model that contains the NMP zeros of the plant, if any. The two-step procedure can then be described as follows.
Step 1. Minimize $\tilde{J}_0^{VR}(\eta,\rho)$. If you want to perform a pole assignment, let the entire numerator of $M(z,\eta)$ free. If not, let some parameters free in order to identify the NMP zeros, if any. Call $(\hat{\eta},\hat{\rho})$ the minimizing parameters and check the step response of $M(z,\hat{\eta})$. If it is satisfactory, apply $C(z,\hat{\rho})$ to the system. If not, go to Step 2.
Step 2. If $M(z,\hat{\eta})$, obtained in Step 1, has NMP zeros, then modify the reference model $\bar{M}(z)$ so that it contains these NMP zeros. If not, keep the initially chosen $\bar{M}(z)$. Then, apply the standard VRFT with $\bar{M}(z)$.

5 Illustrative examples

In this section we present simulation studies using the flexible VRFT scheme with the two-step procedure.

5.1 Process with one non-minimum phase zero

Consider that the process

$$G_1(z) = \frac{(z-1.2)(z-0.4)}{z(z-0.3)(z-0.8)} \tag{20}$$

is controlled by the PID controller

$$C(z,\rho) = \rho^T\beta(z) = [\rho_1\ \rho_2\ \rho_3]\left[\frac{z^2}{z^2-z}\ \frac{z}{z^2-z}\ \frac{1}{z^2-z}\right]^T. \tag{21}$$

A batch of data is obtained from a closed loop experiment, where the reference signal is a sequence of steps and the controller is $C_{init}(z) = \frac{-0.7(z-0.4)(z-0.6)}{z^2-z}$.

5.1.1 Assumption 2 is satisfied

Consider the following desired reference model, which has been chosen in the absence of any knowledge on the NMP zero of $G_1(z)$: $\bar{M}(z) = \frac{0.0706z^2}{(z-0.885)(z^2-0.706z+0.32)}$. The standard VRFT criterion used with this reference model yields the controller

$$C(z,\hat{\rho}) = \frac{-2.269(z^2-1.655z+0.7007)}{z^2-z},$$

which causes the closed loop to be unstable, as can be seen in the corresponding closed loop transfer function

$$T(z,\hat{\rho}) = \frac{-2.2693(z-1.200)(z-0.4000)(z^2-1.655z+0.7007)}{(z-0.3909)(z^2-1.774z+0.7905)(z^2-2.204z+2.470)}.$$

The system instability is due to the NMP zero present in $G_1(z)$ but not in the reference model $\bar{M}(z)$. Note, however, that it is not caused by an unstable pole-zero cancellation between a controller pole and the NMP zero of $G_1(z)$. We thus use the two-step procedure with the following flexible reference model, which satisfies Assumption 2:

$$M(z,\eta) = \frac{\eta_1 z^2+\eta_2 z+\eta_3}{(z-0.885)(z^2-0.706z+0.32)}. \tag{22}$$

This flexible reference model has the same poles as the desired reference model. We minimize $\tilde{J}_0^{VR}(\eta,\rho)$ w.r.t η and ρ using the iterative procedure (16)-(17). The values of $M(z,\hat{\eta}^{(30)})$ and $C(z,\hat{\rho}^{(30)})$ at iteration 30 are as follows:

$$M(z,\hat{\eta}^{(30)}) = \frac{-0.590837(z-1.199909)(z-0.402214)}{(z-0.885)(z^2-0.706z+0.32)},$$
$$C(z,\hat{\rho}^{(30)}) = \frac{-0.590263(z-0.800034)(z-0.300411)}{z^2-z}.$$

Observe that $M(z,\hat{\eta}^{(30)})$ reproduces both zeros of $G_1(z)$ with high precision, and the controller $C(z,\hat{\rho}^{(30)})$ is such that its zeros cancel the poles of the process. A good estimate of the NMP zero is already obtained at iteration $i = 21$, while convergence to the minimum phase zero is slower. This observation is consistent with the findings of [14] where it is shown that NMP zeros are easier to estimate than minimum phase zeros. Since step 1 shows that the process actually has a NMP zero at $z = 1.199909$, we change $\bar{M}(z)$ to include this NMP zero and then use the standard VRFT. That is, a new reference model is defined as

$$\bar{M}_m(z) = \frac{-0.353195(z-1.199909)z}{(z-0.885)(z^2-0.706z+0.32)}, \tag{23}$$

where the gain is chosen so that $\bar{M}_m(1) = 1$. The standard VRFT method using (23) yields the controller

$$C(z,\hat{\rho}) = \frac{-0.451638(z-0.803330)(z-0.097992)}{z^2-z}.$$

Fig. 1 shows the step responses obtained at the end of Step 2 with $T(z,\hat{\rho})$ where $\hat{\rho}$ minimizes (7), as well as the step response of $\bar{M}_m(z)$. Observe that the responses of $T(z,\hat{\rho})$, obtained at the end of Step 2, and $\bar{M}_m(z)$ are very similar. Observe also that Step 2 has led to a closed loop response which is closer to the first desired reference model $\bar{M}(z)$, presenting a smaller inverse response compared to the one obtained at the end of Step 1.

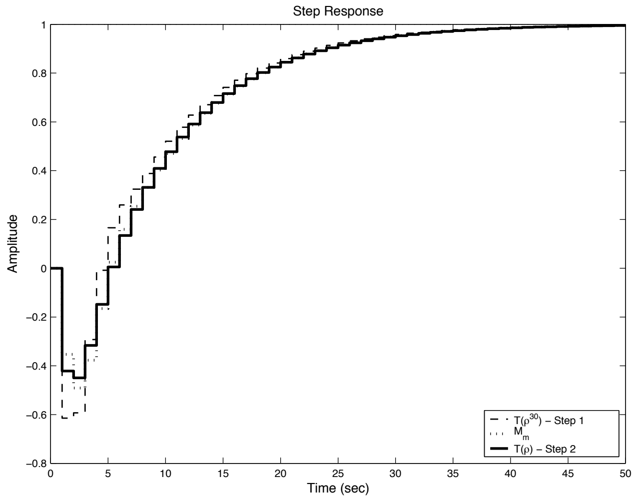

Fig. 1. Closed loop responses $T(z,\hat{\rho})$ obtained at the end of Step 1 and 2, compared to the modified reference model (23).

5.1.2 *Assumption 2 is not satisfied*

In the previous example, the reference model (22) was chosen in such a way that the matching condition (11) is satisfied for some (η^*,ρ^*) pair. Since the process is normally unknown, Assumption 2 can typically not be satisfied. We now examine how the method behaves in this situation.

For the same process (20) we choose a faster fixed reference model $\bar{M}_f(z) = \frac{0.064z^2}{(z-0.6)^3}$, as well as a flexible one defined as $M_f(z,\eta) = \frac{\eta_1 z^2+\eta_2 z+\eta_3}{(z-0.6)^3}$, for which Assumption 2 is not satisfied. We minimize $\tilde{J}_0^{VR}(\eta,\rho)$ w.r.t η and ρ using the iterative procedure (16)-(17). The values of $M(z,\hat{\eta}^{(30)})$ and $C(z,\hat{\rho}^{(30)})$ at iteration 30 are as follows:

$$M_f(z,\hat{\eta}^{(30)}) = \frac{-0.542229(z-1.197399)(z-0.402066)}{(z-0.6)^3},$$
$$C(z,\hat{\rho}^{(30)}) = \frac{-0.523361(z-0.793241)(z+0.009140)}{z^2-z}.$$

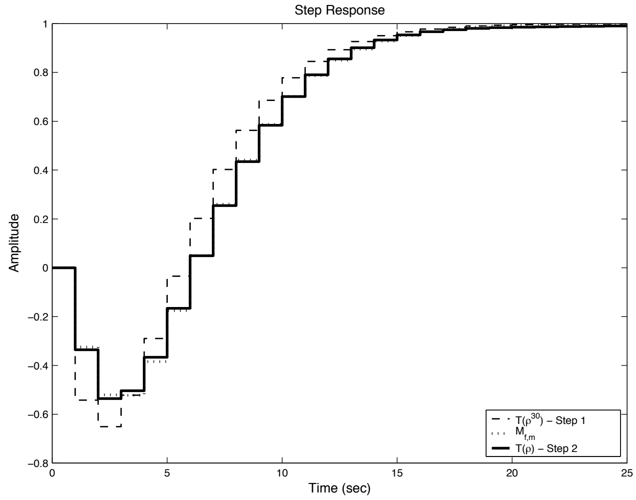

Fig. 2. Closed loop responses $T(z,\hat{\rho})$ obtained at the end of Step 1 and 2, compared to the modified reference model $\bar{M}_{f,m}(z)$.

Indeed, note that, even though Assumption 2 is not satisfied, the NMP zero is still identified with good precision by the minimization of $\tilde{J}_0^{VR}(\eta,\rho)$. We again apply the second step of our procedure, modifying the fixed reference model to include the NMP zero just identified: we choose $\bar{M}_{f,m}(z) = \frac{-0.324216z(z-1.197399)}{(z-0.6)^3}$. Utilizing the standard VRFT procedure, we find the following controller

$$C(z,\rho) = \frac{-0.335024(z+0.564947)(z-0.806625)}{z^2-z}.$$

Fig. 2 presents the modified reference model $\bar{M}_{f,m}(z)$ and the step responses obtained in Step 1 and Step 2. Even though Assumption 2 is not satisfied, and even though the NMP zero imposed on the modified reference model $\bar{M}_{f,m}(z)$ is only approximately correct, the achieved closed loop response is very close to $\bar{M}_{f,m}(z)$. We also observe that the response obtained in Step 2 presents again a smaller inverse response compared to the one obtained at the end of Step 1.

5.2 *Process with two minimum-phase zeros*

Finally, we apply the method to an example in which the plant zeros are both minimum phase:

$$G_2(z) = \frac{(z+0.2)(z-0.4)}{z(z-0.3)(z-0.8)}. \tag{24}$$

This process is initially in closed loop with a PID controller $C_{init}(z) = \frac{0.7(z-0.4)(z-0.6)}{z^2-z}$, which we want to retune so that the closed loop response is as close as possible to a given $\bar{M}(z)$, using a PID controller $C(z,\rho)$ of the form (21).

5.2.1 *Assumption 2 is satisfied*

The desired fixed reference model is given by $\bar{M}(z) = \frac{0.4601z^2}{(z-0.6673)(z^2+0.3063z+0.0766)}$, and the flexible reference model is chosen as

$$M(z,\eta) = \frac{\eta_1 z^2 + \eta_2 z + \eta_3}{(z-0.6673)(z^2+0.3063z+0.0766)},$$

for which Assumption 2 is satisfied. After 40 iterations we obtain

$$M(z,\hat{\eta}^{(40)}) = \frac{0.668120(z-0.415691)(z+0.178555)}{(z-0.6673)(z^2+0.3063z+0.0766)},$$
$$C(z,\hat{\rho}^{(40)}) = \frac{0.667786(z-0.802044)(z-0.306312)}{z^2-z}.$$

Since $M(z,\hat{\eta}^{(40)})$ does not have a NMP zero, we can safely go to Step 2 and use the standard VRFT method without modifying the reference model. The controller obtained with $\bar{M}(z)$ is

$$C(z,\hat{\rho}) = \frac{0.462536(z-0.299652)(z-0.761272)}{z^2-z}.$$

5.2.2 *Assumption 2 is not satisfied*

Suppose now that we choose another fixed reference model: $\bar{M}_f(z) = \frac{0.216z^2}{(z-0.4)^3}$, and a flexible model having the same poles as $\bar{M}_f(z)$: $M_f(z,\eta) = \frac{\eta_1 z^2+\eta_2 z+\eta_3}{(z-0.4)^3}$. With $M_f(z,\eta)$ and the controller (21), Assumption 2 is not satisfied. Then Step 1 leads to

$$M_f(z,\hat{\eta}^{(10)}) = \frac{0.498641(z-0.693423)(z+0.412949)}{(z-0.4)^3},$$
$$C_f(z,\hat{\rho}^{(10)}) = \frac{0.495592(z-0.681493)(z+0.203921)}{z^2-z}.$$

Since the first step computes not only the numerator coefficients of the flexible reference model, but also a corresponding controller, one might consider applying this controller $C_f(z,\hat{\rho}^{(10)})$ to the plant, thereby avoiding the need for a second step. Notice, however, that $T(z,\hat{\rho}^{(10)})$ is far from the desired $\bar{M}_f(z)$ as shown by their step responses in Fig. 3. Thus, the performance produced by the controller $C_f(z,\hat{\rho}^{(10)})$ is far from the desired performance specified by the original reference model $\bar{M}(z)$, as illustrated in the top part of Fig. 3. We thus proceed to Step 2 and apply the standard VRFT with the fixed reference model $\bar{M}_f(z)$. The controller obtained is

$$C(z,\hat{\rho}) = \frac{0.193075(z+0.599708)(z-0.789037)}{z^2-z}.$$

The closed loop response obtained with $C(z,\hat{\rho})$ is compared with that of the fixed reference model $\bar{M}_f(z)$ in the bottom part of Fig. 3.

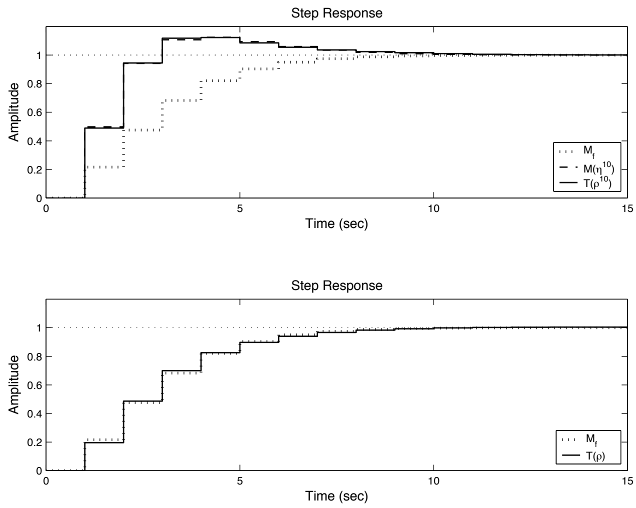

Fig. 3. Step responses of the fixed reference model $\bar{M}_f(z)$, the flexible reference model $M(z,\hat{\eta}^{(10)})$ and the closed loop response $T(z,\hat{\rho}^{(10)})$ obtained after 10 iterations in Step 1 (top figure); step responses of the original fixed reference model $\bar{M}_f(z)$ and of the closed loop system $T(z,\hat{\rho})$ obtained in Step 2.

6 Conclusions and future work

We have extended the VRFT design methodology to cope with NMP plants. This has been achieved through a flexible design criterion, in which the numerator of the reference model is left free to be adjusted to the zeros of the plant through the optimization procedure. We have proposed a two-step procedure in which the possible presence of NMP zeros in the plant, as well as their location, is detected in the first step. The first step yields a reference model and a controller, and can be used as the solution of a pole assignment problem. If the closed loop response obtained in step one is not satisfactory, the designer can still perform the step 2, which becomes a classical VRFT, but with a criterion that takes into account the presence of the NMP zeros, if any, detected in the first step.

Relevant properties of the method have been established through theoretical analysis. Extending this analysis to the case where the matching controller is not in the controller class, as well as adapting the method to deal with signals corrupted by noise, are some of the aims of our future research. Simulations have already shown good performance for the situation where the matching condition is not satisfied.

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