

Tuning of multivariable decentralized controllers through the Ultimate Point Method

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Abstract—A successful method for tuning single-loop PID controllers is the Ultimate Point Method. This method is based on the identification of the ultimate point of the process’ frequency response followed by its shifting, through appropriate choices of the controller’s parameters, to a specified location of the complex plane. This paper presents an extension of the Ultimate Point Method to the tuning of decentralized controllers for multivariable processes.

Index Terms—Ultimate point method, PID control, multivariable decentralized control, relay feedback experiment.

I. INTRODUCTION

Despite several decades of industrial PID control practice, a large proportion of the control loops still presents poor performance [1]. For many control loops, a good tuning method is one that would require little information about the process dynamics, so that this information can be obtained by carrying out an experiment of limited complexity, followed by the application of simple tuning rules. One of the most successful methods is the one based on the knowledge of the ultimate point of the process’ frequency response, that is, the point at which its phase reaches -180° . The characteristics of the frequency response at this point are usually called the ultimate quantities of the process: the ultimate period and gain. In SISO (single-input, single-output) control, well-known formulae, such as Ziegler-Nichols and Tyreus-Luyben, use these ultimate values in order to tune PI and PID controllers [1], [2], [3]. Different names have been given to this method; in this paper we refer to it as the *Ultimate Point Method*.

In multivariable (MIMO - multiple-input, multiple-output) processes, it is common practice to use the SISO procedure and tuning formulae, followed by a “detuning”, which consists in reducing the controller gains by some ad hoc factor [4]. This gain reduction tends to enhance the SISO stability margins, so that the coupling with other loops will - hopefully - not destabilize the loop. Better tuning can be obtained by using sequential controller tuning [2], [5], but this procedure may require a large number of experiments. Other techniques have been applied as well, mostly based on the SISO method and experiments [6], [4]. Other methods deal with the identification of more than one point of the frequency response followed by

the controller tuning [7], [8], but these usually apply to cross-coupled (that is, not decentralized) controllers. Also, there are tuning methods that yield more efficient controllers, but are much more demanding in terms of the data that must be collected from the process’ operation [9].

More recently, extensions of the Ultimate Point Method for decentralized control of multivariable processes have been proposed, [10], [11], [12]. In these papers, the multivariable nature of the problem is explicitly acknowledged: the MIMO ultimate quantities are obtained by means of multivariable experiments. Yet, this information is used for tuning the controllers with the SISO formulae, a procedure that does not seem substantiated by formal multivariable analysis. Moreover, MIMO processes have infinitely many ultimate points, and the resulting closed-loop performance is strongly related to the particular ultimate point used. Hence, a better understanding of the MIMO control problem and how to use the ultimate quantities to design MIMO controllers is in order [13], [14].

In this paper we provide a consistent criterion for determining the settings of decentralized PID controllers based on the ultimate quantities identified. The tuning is made based on a multivariable frequency response criterion: shifting the ultimate point to an assigned location in the complex plane. In so doing, like in the SISO case, adequate stability margins can be obtained, provided that the frequency response of the process satisfies certain properties. The closed-loop performance obtained with this method presents little sensitivity to the particular ultimate point used for the design.

We consider MIMO square processes described by a transfer matrix

$$Y(s) = G(s)U(s) \quad (1)$$

where $Y(s)$ and $U(s)$ are vectors with m components, representing the Laplace transforms of the process output and input respectively, and $G(s)$ is the process’ transfer matrix. The processes are assumed to be BIBO-stable.

We aim at designing decentralized controllers

$$U(s) = C(s)E(s) \quad (2)$$

$$= \begin{bmatrix} p_1(s) & 0 & 0 & \dots & 0 \\ 0 & p_2(s) & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & \dots & p_m(s) \end{bmatrix} \begin{bmatrix} e_1(s) \\ e_2(s) \\ \vdots \\ e_m(s) \end{bmatrix}$$

where $E(s) = R(s) - Y(s)$, $R(s)$ is the Laplace transform of the reference signal, and $p_i(s)$, $i = 1, \dots, m$ are the - scalar - transfer functions of the decentralized controller $C(s)$. We deal with PID controllers, in which $p_i(s) = k_{pi} + \frac{k_{ii}}{s} + k_{di}s$, with k_{pi} , k_{ii} , $k_{di} \in \mathfrak{R}$. Although in this paper we focus on PID

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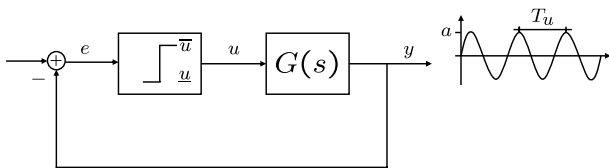


Fig. 1. Relay feedback experiment.

controllers, the method can be applied to set the parameters of any fixed structure controller.

The outline of the paper is as follows. Section II presents the Ultimate Point Method for SISO systems and Section III presents how the ultimate quantities are defined in MIMO processes. The Multivariable Ultimate Point Method is presented in Section IV. As the method needs the knowledge of the process' frequency response at a given frequency, Section V discusses the practical determination of this quantity. The method is illustrated by two examples: in Section VI some results of its application to a benchmark - the Wood and Berry distillation column - are given and an additional, illustrative example, is discussed in Section VII. Finally, Section VIII presents the conclusions.

II. ULTIMATE POINT METHOD

The Ultimate Point Method for tuning SISO PID controllers is a well-known method based on the knowledge of the ultimate point of the process' frequency response, which is the point at which the process' Nyquist diagram intersects the negative real axis of the complex plane [1]. Associated with this point are the *ultimate quantities*: the ultimate gain (K_u) and ultimate period (T_u) of the process. If the process is put under purely proportional control the ultimate gain is the gain for which the loop is at its stability boundary. The ultimate frequency (ω_u) is the frequency at which the crossing of the negative real axis happens, and the ultimate period is given by $T_u = 2\pi/\omega_u$. The ultimate quantities are more conveniently obtained by the relay feedback experiment [1]. This experiment consists of a closed-loop bang-bang control, as shown in Fig. 1. The control function $\eta(\cdot)$ is described as

$$u = \eta(e) = -d \operatorname{sign}(e) + \operatorname{bias} \quad (3)$$

where the bias term should be chosen as $\operatorname{bias} = r/G(0)$ and the relay amplitude is given by $d = (\underline{u} - \bar{u})/2$ [1]. Then, if the ultimate point does exist, a symmetric oscillation will be observed, and this oscillation presents the properties below.

Fact 1: [1] Let ω_{osc} be the frequency of the oscillation observed in the relay experiment and a be its amplitude at the relay input. Since the ultimate frequency is such that $\angle G(j\omega_u) = -\pi$, ω_{osc} gives an estimate for the ultimate frequency, so that both ultimate quantities can be obtained by means of the relay experiment. Then

$$\omega_u = \omega_{osc} \quad (4)$$

$$K_u = \frac{4d}{\pi a} \quad (5)$$

Once the ultimate quantities are determined, the PID is tuned according to given formulae. Different sets of tuning

TABLE I
ZIEGLER-NICHOLS AND TYREUS-LUYBEN FORMULAE.

Ziegler-Nichols	k_p	T_i	T_d
PI	$0.4K_u$	$0.8T_u$	0
PID	$0.6K_u$	$0.5T_u$	$0.125T_u$
Tyreus-Luyben	k_p	T_i	T_d
PI	$K_u/3.2$	$2.2T_u$	0
PID	$K_u/2.2$	$2.2T_u$	$T_u/6.3$

TABLE II
DIFFERENT POINTS TO WHICH THE ULTIMATE POINT IS MOVED USING ZIEGLER-NICHOLS AND TYREUS-LUYBEN FORMULAE

Ziegler-Nichols	PI	$-0.4 + 0.08j$
	PID	$-0.6 - 0.28j$
Tyreus-Luyben	PI	$-0.31 + 0.023j$
	PID	$-0.45 - 0.42j$

rules have been proposed over the years, aiming at different performance criteria. We will explore two sets of formulae in this paper: the originally proposed Ziegler-Nichols formulae [3], which typically provide fast but oscillatory behavior, and the more conservative Tyreus-Luyben formulae [15]. These formulae are given in Table I where the transfer function of the PID controller is given by

$$C(s) = k_p \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (6)$$

The PID controller dislocates the ultimate point into a stable region, which is away from the point $-1 + 0j$ and closer to the origin of the complex plane. Each set of formulae corresponds to moving the ultimate point to a given location in the complex plane. A PI controller with Ziegler-Nichols tuning, for example, has $k_p = 0.4K_u$ and $\omega_u T_i = (2\pi/T_u)0.8T_u = 5.02$. Then, the PI controller transfer function, at the ultimate frequency is

$$\begin{aligned} C(j\omega_u) &= k_p \left(1 + \frac{1}{j\omega_u T_i} \right) = 0.4K_u \left(1 - \frac{j}{5.02} \right) \\ &= K_u (0.4 - 0.08j) \end{aligned}$$

which results in the loop transfer function

$$L(j\omega_u) = C(j\omega_u)G(j\omega_u) = -0.4 + 0.08j.$$

That is, the ultimate point has been moved to $-0.4 + 0.08j$. Table II shows the points to which different formulae move a process ultimate point.

Stability margins

From the Nyquist stability criterion it follows that if the whole frequency response is kept away from -1 then the closed-loop system will be stable and present adequate stability (gain and phase) margins. This means that if the ultimate point is away from -1 and closer to the origin, nearby points of the frequency response will also be away and, provided that the frequency response is "sufficiently smooth", adequate stability margins are obtained. Different smoothness assumptions on the frequency response have been made in order to associate stability margins to the Ultimate Point Method [1], [16], [2].

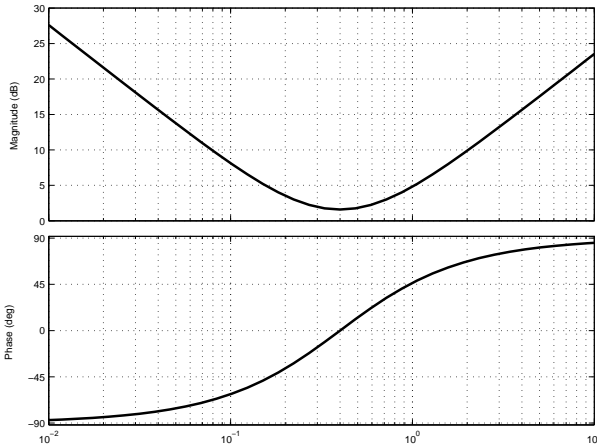


Fig. 2. Bode diagram of a PID controller for a system with $K_u = 2$ and $\omega_u = \frac{2\pi}{10}$ rad/s.

If, for instance, the Ziegler-Nichols tuning is used in a PID controller, then

$$\begin{aligned} C(s) &= 0.6K_u \left(1 + \frac{1}{s\pi/\omega_u} + \frac{0.25\pi s}{\omega_u} \right) \\ &= \frac{0.15\pi K_u (s + \frac{2}{\pi}\omega_u)^2}{s} \end{aligned} \quad (7)$$

That is, the controller presents a double zero at the frequency $-\frac{2}{\pi}\omega_u$, at which frequency its magnitude is minimal and equal to $|C(j\frac{2}{\pi}\omega_u)| = 0.6K_u$. Hence the frequency response of the controller has the form depicted in Figure 2.

Stability in closed-loop is obtained if the loop transfer function $C(j\omega)G(j\omega)$ satisfies

$$|C(j\omega_1)G(j\omega_1)| < 1 \quad (8)$$

$$\angle C(j\omega_0)G(j\omega_0) \geq -180^\circ \quad (9)$$

where ω_1 and ω_0 are defined as $\angle C(j\omega_1)G(j\omega_1) = -\pi$ and $|C(j\omega_0)G(j\omega_0)| = 1$. With the Ziegler-Nichols tuning we have by definition $C(j\omega_u)G(j\omega_u) = -0.6 - 0.28j = 0.66 \angle -25.02^\circ$. If the process' module drops faster than the growth of the controller's module for all frequencies above ω_u and its phase drops faster than the growth of the controller's phase for all frequencies below ω_u , then it is clear that conditions (8) and (9) will be satisfied. We have thus established the following fact.

Fact 2: Let $G(s)$ be the transfer function of a BIBO-stable SISO process and let this process be controlled by a PID controller (6) with the Ziegler-Nichols tuning as given in Table I. If the Bode plot of the process' frequency response lies completely outside the shaded area in Figure 3 then the closed-loop system is BIBO-stable.

The constraints on the frequency response of the process in the fact above have clear physical interpretation. Although these conditions can usually not be directly checked for a given process (checking them would require a rough model of the process) they give a guideline to the class of processes to which Ziegler-Nichols tuning can be successfully applied. For instance, a delay-free stable transfer function with no zeros will always satisfy these constraints provided that its poles

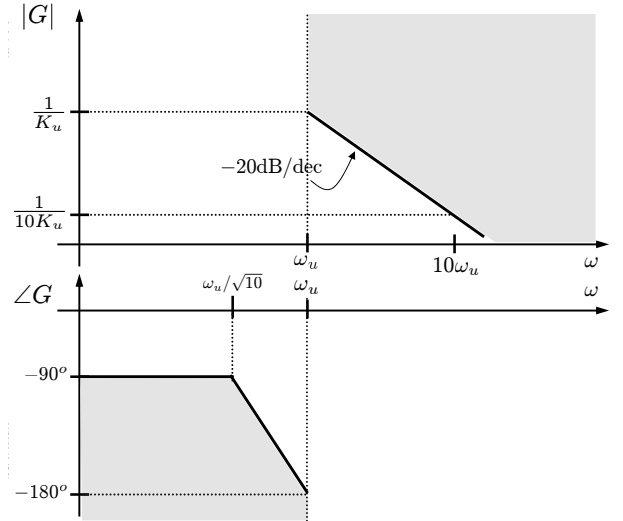


Fig. 3. Stability boundaries for the process' frequency response

are not too far apart from each other. On the other hand, several classes of processes are excluded, such as systems with dominant time-delay and oscillatory systems with natural oscillation frequency close to the ultimate frequency. For these systems stability can not be guaranteed and for many among them the closed-loop system will indeed be unstable, if a Ziegler-Nichols tuning is applied.

Different sufficient conditions, with different degrees of conservatism, can be obtained for the Ziegler-Nichols tuning, as well as for other tuning rules [1], [16]. The same considerations will apply regarding the closed-loop stability when applying the Multivariable Ultimate Point Method.

III. ULTIMATE QUANTITIES IN MIMO PROCESSES

In multivariable processes like (1) the ultimate quantities can be defined similarly to the SISO case [14].

Definition 1: Let a BIBO-stable square process with m inputs be controlled by purely proportional controller $u = -Ky$, with gain $K = \text{diag}\{k_1 \ k_2 \ \dots \ k_m\}$, $k_i \in [0, \infty)$. Since the process is BIBO-stable, the feedback system is BIBO-stable for sufficiently small K . Assume that there exists a constant matrix K_u such that the closed-loop system is BIBO-stable $\forall K = \alpha K_u$, $0 < \alpha < 1$ and unstable for $K = K_u(1 + \epsilon)$, with ϵ an arbitrarily small positive scalar; this value K_u is called an *ultimate gain* of the process. On the other hand, for $K = K_u$ the closed-loop system is on the verge of stability and hence a sustained oscillation will be observed; the frequency of this oscillation is called an *ultimate frequency* ω_u . \square

The stability of MIMO square systems can be analysed through the Nyquist criterion, as in the SISO case. Denote $\lambda_i(s)$, $i = 1, 2, \dots, m$ the eigenvalues of $G(s)K$. The graphs of $\lambda_i(s)$ as s goes round the Nyquist contour are called the characteristic loci. With these definition, the Nyquist theorem for MIMO systems can be stated as follows.

Theorem 1 (Generalized Nyquist theorem): [8] If $G(s)$ has P_0 unstable (Smith-McMillan) poles, then the closed-loop system with return ratio $-G(s)K$ is stable if and only if the

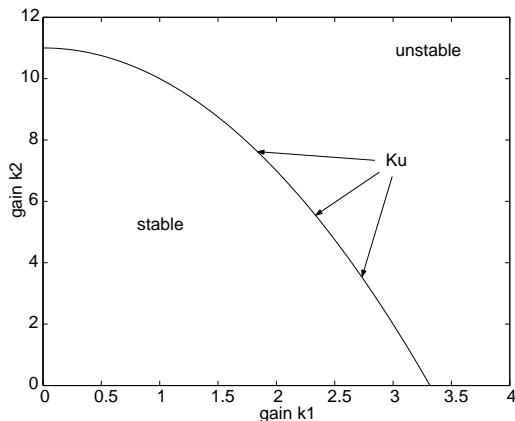


Fig. 4. The ultimate gains in the TITO (two-input, two-output) case form a curve in the parameter space.

characteristic loci of $G(s)K$, taken together, encircle the point -1 P_0 times counterclockwise, assuming that there are no hidden unstable modes. \square

So, for a BIBO-stable $G(s)$, the closed-loop system is stable if and only if none of the characteristic loci of $G(s)K$ encircles the point -1 . As the gain matrix K is varied, the stability limit is reached when at least one of the characteristic loci equals -1 for some frequency. The gain for which this happens is an ultimate gain.

The ultimate quantities are usually unique in the SISO case; even when they are not unique, they are countable. The situation is quite different in the MIMO case, since the gain matrix K can be increased from 0 in infinite different directions in the parameter space. It can be expected that a different K_u and ω_u will be found for each different direction, as depicted in Fig. 4. The set of all the ultimate gains is a curve in the parameter space. In the more general ($m > 2$) case these gains will form a surface of dimension $m-1$; this surface will be called the *ultimate surface* [14].

If decentralized PID tuning is determined based on the ultimate quantities using Ziegler-Nichols like formulae, as in [10], [11], [12], [17], then two things must be realized. First, all PID's will be tuned based on the same ultimate frequency. Second, the tuning will be dependent on which pair of the ultimate quantities has been identified.

For simplicity, let $m = 2$. Consider that the gain K is increased in the direction $K = \text{diag}\{k_1 \ 0\}$, that is, the second loop is kept open, and the proportional gain in loop 1 is increased. The ultimate gain that will be obtained in this experiment is the SISO ultimate gain k_{u1} of the first loop. Then, if the PID is tuned according to these ultimate quantities, this tuning is the “correct” one for the first SISO loop. On the other hand, if the gain K is increased as $K = \text{diag}\{0 \ k_2\}$, then the “correct” SISO tuning for the second loop is obtained. If a different direction in the parameter space is picked to increase the gain K , then it is expected to find a tuning that will not be optimal for any of the two loops, but will represent some sort of “average” of the two. The closer this direction is to either one of the two SISO directions defined above, the closer the

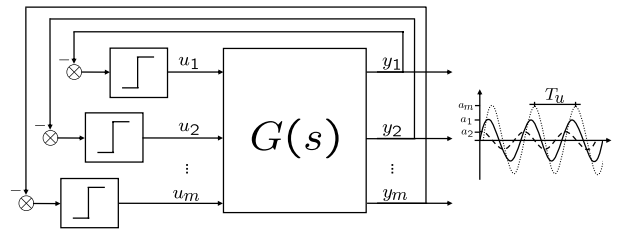


Fig. 5. Decentralized relay feedback experiment.

tuning is expected to be to the corresponding “correct” SISO tuning for that loop [10].

Once the ultimate quantities are identified, a gain sufficiently smaller than the ultimate one should guarantee stability and enough stability margins. However, this is not necessarily true in the MIMO case. If the ultimate curve in the parameter space is convex, then adequate stability margins are guaranteed by taking a gain $K = \alpha K_u$ for sufficiently small α . However, it is not possible to guarantee that the ultimate surface is convex, or even smooth. As a matter of fact, there are cases in which the ultimate surface is not convex, like in the benchmark studied in Section VI.

The ultimate gains can be more conveniently obtained by a Decentralized Relay Feedback (DRF) experiment [10], illustrated in Fig. 5, and different ultimate points are identified with different relay amplitudes [13], [14]. In DRF only one experiment is performed, with all control loops in relay feedback, that is $u_i = \eta(e_i) \forall i$. Since all the input-output pairs are connected, the behavior of the whole multivariable system is observed in this single experiment.

IV. DESIGN BASED ON THE MIMO ULTIMATE QUANTITIES

The Multivariable Ultimate Point Method is an extension of the Ultimate Point Method to multivariable systems. The main idea is to dislocate the ultimate point of the process to another point in the complex plane, moving it to a position that is away from the point $-1 + 0j$ and closer to the origin [13]. Let us first present the TITO (two-input-two-output) case.

A. The TITO case

Consider a 2×2 or TITO system. Then the loop transfer function for $K = K_u$ is given by:

$$G(j\omega)K_u = \begin{bmatrix} g_{11}(j\omega) & g_{12}(j\omega) \\ g_{21}(j\omega) & g_{22}(j\omega) \end{bmatrix} \begin{bmatrix} k_{u1} & 0 \\ 0 & k_{u2} \end{bmatrix} \quad (10)$$

Let $\lambda_1(j\omega)$ and $\lambda_2(j\omega)$ be the eigenvalues of the loop transfer matrix $G(j\omega)K_u$. Then, by the very definition of critical gain K_u and critical frequency ω_u , at least one of these eigenvalues is equal to -1 for $\omega = \omega_u$, that is, either $\lambda_1(j\omega_u) = -1$ or $\lambda_2(j\omega_u) = -1$ or both. Replacing the proportional controller with a decentralized dynamic controller in (10), we have

$$G(j\omega)C(j\omega) = \begin{bmatrix} g_{11}(j\omega) & g_{12}(j\omega) \\ g_{21}(j\omega) & g_{22}(j\omega) \end{bmatrix} \begin{bmatrix} p_1(j\omega) & 0 \\ 0 & p_2(j\omega) \end{bmatrix} \quad (11)$$

The ultimate point is to be dislocated to a previously chosen position in the complex plane, that is, at the ultimate

frequency ω_u , the eigenvalues of the loop matrix in (11) must be located at these previously chosen locations. From (11), the characteristic loci at the ultimate frequency are given by

$$\det(\lambda(j\omega_u)I - G(j\omega_u)C(j\omega_u)) = 0. \quad (12)$$

The ultimate point design consists in finding controller's transfer functions $p_1(s)$ and $p_2(s)$ such that the eigenvalues $\lambda(j\omega_u)$ in (12) have the desired values, say Λ_1 and Λ_2 ; this is the core aspect of the MIMO ultimate point method. Let us give a more detailed treatment for a particular way of choosing these locations in the complex plane, namely $\Lambda_1 = \Lambda_2 = \Lambda$. With this choice, expanding (12), and defining $g_{11_u} = g_{11}(j\omega_u)$, $g_{12_u} = g_{12}(j\omega_u)$, $g_{21_u} = g_{21}(j\omega_u)$, $g_{22_u} = g_{22}(j\omega_u)$, $p_{1_u} = p_1(j\omega_u)$, $p_{2_u} = p_2(j\omega_u)$ in order to simplify the notation, we have

$$\begin{aligned} \lambda^2 - (g_{11_u}p_{1_u} + g_{22_u}p_{2_u})\lambda + \\ (g_{11_u}g_{22_u} - g_{12_u}g_{21_u})p_{1_u}p_{2_u} = 0 \end{aligned} \quad (13)$$

If the roots of (13) are given by Λ , then the characteristic equation is given by $\lambda^2 - 2\Lambda\lambda + \Lambda^2$, which yields

$$2\Lambda = g_{11_u}p_{1_u} + g_{22_u}p_{2_u} \quad (14)$$

$$\Lambda^2 = (g_{11_u}g_{22_u} - g_{12_u}g_{21_u})p_{1_u}p_{2_u} \quad (15)$$

From (14), we have

$$p_{2_u} = \frac{2\Lambda}{g_{22_u}} - \frac{g_{11_u}}{g_{22_u}}p_{1_u}, \quad (16)$$

and, substituting (16) into (15), we have

$$\frac{g_{11_u}}{g_{22_u}}p_{1_u}^2 - \frac{2\Lambda}{g_{22_u}}p_{1_u} + \frac{\Lambda^2}{g_{11_u}g_{22_u} - g_{12_u}g_{21_u}} = 0 \quad (17)$$

Since (17) is a second order polynomial, we can get two different tunings for p_{1_u} . Note that p_{1_u} is a complex quantity. Then, by solving (17) we have the setting of the tunings for p_{1_u} and, substituting into (16), two different tunings for p_{2_u} , also complex, are obtained. Therefore, we have two different controllers that move the ultimate point to the desired location.

For PI controllers, from the solution of (17), we have

$$k_{p1} = \operatorname{Re}\{p_{1_u}\}, \quad (18)$$

$$k_{i1} = -\operatorname{Im}\{p_{1_u}\}\omega_u, \quad (19)$$

and p_{2_u} gains are given by

$$k_{p2} = \operatorname{Re}\{p_{2_u}\}, \quad (20)$$

$$k_{i2} = -\operatorname{Im}\{p_{2_u}\}\omega_u. \quad (21)$$

For PID controllers, we obtain p_{1_u} in the same way (from (17)), but in this case

$$p_{1_u} = k_{p1}\left(1 + (T_{d1}\omega_u - \frac{1}{T_{i1}\omega_u})j\right).$$

and we have

$$k_{p1} = \operatorname{Re}\{p_{1_u}\}, \quad (22)$$

$$k_{p1}(T_{d1}\omega_u - \frac{1}{T_{i1}\omega_u}) = \operatorname{Im}\{p_{1_u}\}. \quad (23)$$

One degree of freedom still exists in the choice of T_{i1} and T_{d1} . A possible choice is to set $T_{d1} = \frac{T_{i1}}{4}$, which is usual

also in SISO design, causing the two zeros of the controller to be equal, which guarantees smoothness of the controller's frequency response. In this case, (23) becomes

$$k_{p1}(T_{i1}\omega_u)^2 - 4\operatorname{Im}\{p_{1_u}\}T_{i1}\omega_u - 4k_{p1} = 0 \quad (24)$$

Thus, from (24) we have two solutions for $T_{i1}\omega_u$, which are always real and with opposite signs. We choose the solution where $T_{i1}\omega_u > 0$, and from the relations $k_{i1} = k_{p1}/T_{i1}$ and $k_{d1} = k_{p1}T_{i1}/4$ we obtain the gains of the controller p_{1_u} . Controller p_{2_u} is obtained from (16) and then its parameters T_{i2} and k_{p2} from (24) *mutatis mutandis*. As for PI controllers, we find two different PID controllers that dislocate both characteristic loci to the desired point.

B. The general case

For the general MIMO case, where m may be different from two, the same reasoning holds and the same procedure can be applied. Of course, the equations become more complex. Consider again the case in which all characteristic loci are dislocated to the same desired point Λ ; for a TITO process, the controllers are obtained through (14) and (15). For a process of order m , the equations used to obtain the controllers are given by

$$a_{m-i}\Lambda^i = \sum M_i(G(j\omega_u)C(j\omega_u)), \quad (25)$$

where M_i are the i -th order principal minors of the matrix $G(j\omega_u)C(j\omega_u)$ and each sum is taken over all principal minors of order i . We note that in each term there are always $C_m^i = \frac{m!}{i!(m-i)!}$ such minors where C_m^i stands for the combination of m elements taken i at a time.

On the other hand, a_{m-i} are the coefficients of the characteristic equation $(s - \lambda)^m$, which can be obtained using the Newton's binomial formula, for $i = 1 \dots m$. So, we have

$$a_{m-i} = C_m^i = \frac{m!}{(m-i)!i!}. \quad (26)$$

Like in the TITO case, this will result in a system of equations with the controllers' values as unknowns.

We present the case where $m = 3$ for illustration. The coefficients of the characteristic equation are given by (26)

$$\begin{cases} i = 1 \rightarrow a_2\Lambda = C_3^1\Lambda = 3\Lambda, \\ i = 2 \rightarrow a_1\Lambda^2 = C_3^2\Lambda^2 = 3\Lambda^2, \\ i = 3 \rightarrow a_0\Lambda^3 = C_3^3\Lambda^3 = \Lambda^3. \end{cases} \quad (27)$$

Equating (27) with (25) results in the following set of equations:

$$3\Lambda = g_{11_u}p_{1_u} + g_{22_u}p_{2_u} + g_{33_u}p_{3_u} \quad (28)$$

$$\begin{aligned} 3\Lambda^2 = & (g_{11_u}g_{22_u} - g_{12_u}g_{21_u})p_{1_u}p_{2_u} + \\ & (g_{11_u}g_{33_u} - g_{13_u}g_{31_u})p_{1_u}p_{3_u} + \\ & (g_{22_u}g_{33_u} - g_{23_u}g_{32_u})p_{2_u}p_{3_u} \end{aligned} \quad (29)$$

$$\begin{aligned} \Lambda^3 = & [(g_{11_u}g_{22_u}g_{33_u} + g_{21_u}g_{13_u}g_{32_u} + g_{31_u}g_{12_u}g_{23_u}) \\ & - (g_{11_u}g_{23_u}g_{32_u} + g_{21_u}g_{12_u}g_{33_u} + g_{31_u}g_{13_u}g_{22_u})] \\ & p_{1_u}p_{2_u}p_{3_u}. \end{aligned} \quad (30)$$

The controller is obtained by solving (28-30) for the unknowns p_{1u} , p_{2u} and p_{3u} . This results in six different solutions - in contrast to the two solutions for the TITO case - and it is not clear how to choose among them. Other design choices also become significantly more involved for systems of dimension larger than two. The handling of these choices is by no means straightforward, so this is a topic that should be further developed to make our method more practical for systems with more than two inputs, although it is in principle applicable to systems of any dimension.

V. PRACTICAL DETERMINATION OF $G(j\omega_u)$

In the SISO ultimate point method, the design of the controller requires the knowledge of the process' frequency response at the ultimate point, an information that is provided by the relay feedback experiment. Accordingly, the MIMO ultimate point method, as presented in the previous section, requires knowledge of the transfer matrix at the ultimate frequency. However, the decentralized relay feedback (DRF) does not provide the whole transfer matrix, but only the ultimate quantities of the process, which for MIMO processes is not equivalent. There are different ways of determining $G(j\omega_u)$ and the more accurate ones will be more demanding. The aim of this section is to present a convenient procedure to apply in operating systems, although the development of optimized procedures remains an open issue.

Consider a TITO process. When the DRF is applied, $u_1(t)$, $u_2(t)$, $y_1(t)$ and $y_2(t)$ are periodic signals. Let u_1 , u_2 , y_1 and y_2 be the complex coefficients of the first harmonics of these signals. Then we have:

$$\begin{cases} y_1 = g_{11}(j\omega_u)u_1 + g_{12}(j\omega_u)u_2 \\ y_2 = g_{21}(j\omega_u)u_1 + g_{22}(j\omega_u)u_2. \end{cases} \quad (31)$$

As we have two equations and four unknowns, it is not possible to determine $G(j\omega_u)$. So, an additional experiment is necessary. From the DRF test, we know the ultimate frequency ω_u of the system. So, if we apply an input $u_1(t) = \sin(\omega_u t)$ and $u_2(t) = 0$ to the system, we have

$$\begin{cases} y_1 = g_{11}(j\omega_u)u_1 \\ y_2 = g_{21}(j\omega_u)u_1, \end{cases} \quad (32)$$

and it is easy to determine $g_{11}(j\omega_u)$ and $g_{21}(j\omega_u)$. Substituting these quantities in (31), we determine $G(j\omega_u)$. However, this additional experiment is an open-loop test, which is usually inconvenient in real applications. In many situations, it is difficult to apply this signal to the input of the process depending on the software and hardware structure in the control system implementation.

A more convenient solution is to apply a second DRF test on the system, changing one relay's amplitude in such a way that the oscillation frequency does not change significantly. Thus, we have

$$\begin{cases} y_1^I = g_{11}(j\omega_u^I)u_1^I + g_{12}(j\omega_u^I)u_2^I \\ y_2^I = g_{21}(j\omega_u^I)u_1^I + g_{22}(j\omega_u^I)u_2^I \\ y_1^{II} = g_{11}(j\omega_u^{II})u_1^{II} + g_{12}(j\omega_u^{II})u_2^{II} \\ y_2^{II} = g_{21}(j\omega_u^{II})u_1^{II} + g_{22}(j\omega_u^{II})u_2^{II}. \end{cases} \quad (33)$$

where the indices I and II mean first and second DRF tests in the plant and the two oscillating frequencies ω_u^I and ω_u^{II} are close to each other. If we make the approximation $\omega_u^I = \omega_u^{II}$, we have four equations and four unknowns, and we can estimate $\hat{G}(j\omega_u)$ by

$$\begin{aligned} \begin{bmatrix} \hat{g}_{11} & \hat{g}_{12} \\ \hat{g}_{21} & \hat{g}_{22} \end{bmatrix} \begin{bmatrix} u_1^I & u_1^{II} \\ u_2^I & u_2^{II} \end{bmatrix} &= \begin{bmatrix} y_1^I & y_1^{II} \\ y_2^I & y_2^{II} \end{bmatrix} \\ \hat{G}U &= Y \\ \hat{G} &= YU^{-1}, \end{aligned} \quad (34)$$

Having these informations, the Multivariable Ultimate Point Method can be used in order to obtain the controller. The difference between the two frequencies will introduce an error in the estimation of $G(j\omega_u)$ but, as the oscillation frequencies are quite similar, we can expect this error to be small [16]. The case studies presented in the sequel show that this is indeed the case and that this procedure can, in many cases, be used to obtain the matrix $G(j\omega_u)$.

VI. CASE STUDY - DISTILLATION COLUMN

Aiming at demonstrating the use of the proposed design method, this section presents some results obtained for the control of the Wood and Berry distillation column, whose transfer matrix is presented in (35). This process has been widely used as a benchmark [11], [18].

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \quad (35)$$

Since the process is 2×2 , we have infinite different combinations of k_{u1} and k_{u2} that lead the process to the stability limit, each one corresponding to a different ultimate point.

In order to illustrate the results, we have tested different ultimate points. As the Multivariable Ultimate Point Method needs the knowledge of $G(j\omega_u)$, we have to find two different but very close ultimate points and then estimate $\hat{G}(j\omega_u)$ by (34). We have estimated $\hat{G}(j\omega_u)$ for two different frequencies, which are presented in Table III, where

$$\Delta G = \frac{\|G(j\omega_u^I) - G(j\omega_u^{II})\|_2}{\|G(j\omega_u^I)\|_2} \quad (36)$$

is the variation of G due to the difference of the oscillation frequencies obtained in each DRF test and

$$e = \frac{\|G(j\omega_u^I) - \hat{G}(j\omega_u^I)\|_2}{\|G(j\omega_u^I)\|_2}. \quad (37)$$

is the estimation error of $\hat{G}(j\omega_u)$ related to $G(j\omega_u)$, ω_u being obtained from the first DRF test performed.

A. PID controller obtained using the Ziegler-Nichols formulae

This section presents some results obtained for the Wood and Berry distillation column using the ultimate quantities of the process and Ziegler-Nichols formulae. This tuning procedure is adopted in several papers [18], [10], [11]. We show the results for references $r(t) = [1 \ 0]^T$ and $r(t) = [0 \ 1]^T$.

Fig. 6 presents the behavior of the system with a PID controller tuned through the Ziegler-Nichols formulae and

TABLE III
 $\hat{G}(j\omega_u)$ FOR TWO DIFFERENT FREQUENCIES OF WOOD AND BERRY COLUMN.

experiment	$ u_2 / u_1 $	ω_u	$\hat{G}(j\omega_u)$	ΔG	e (%)
1 2	1.9371 0.9489	0.4946 0.4931	$\hat{G}_1 = \begin{bmatrix} -0.5554 - 1.4295j & 1.7820 + 0.3223j \\ 0.1784 + 1.1898j & 2.6252 + 0.5941j \end{bmatrix}$	2.2	0.6
3 4	3.5303 0.6659	0.5228 0.5106	$\hat{G}_2 = \begin{bmatrix} -0.5843 - 1.3663j & 1.7265 + 0.2054j \\ 0.2731 + 1.1438j & 2.5441 + 0.4183j \end{bmatrix}$	7.0	6.6

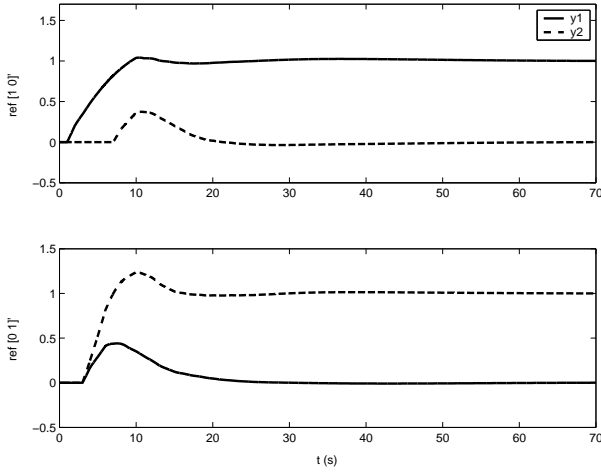
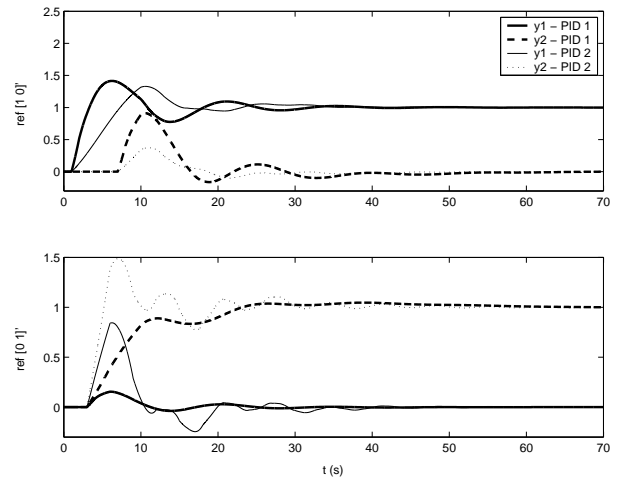
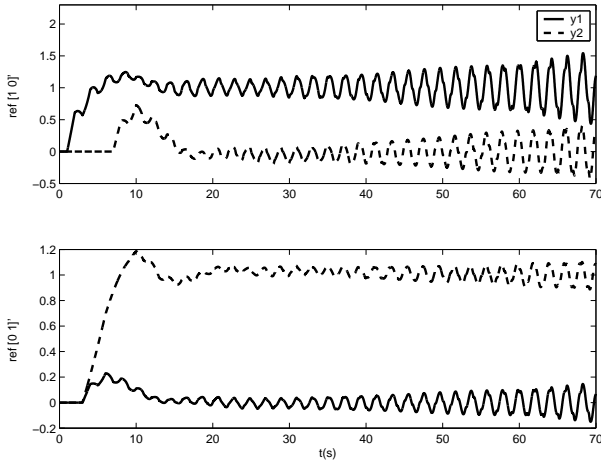

 Fig. 6. Step response of the system with PID tuned using Ziegler-Nichols formulae and ultimate point $K_u = [0.42 \ -0.30]$ and $\omega_u = 0.4882 \text{ rad/s}$.


Fig. 8. Step responses of the system with PID 1 and PID 2, obtained based on Tyreus-Luyben point.


 Fig. 7. Step response of the system with PID tuned using Ziegler-Nichols formulae and ultimate point $K_u = [1.27 \ -0.25]$ and $\omega_u = 0.5129 \text{ rad/s}$.

the ultimate quantities $K_u = [0.42 \ -0.30]$ and $\omega_u = 0.4882 \text{ rad/s}$. Note that the transient performance obtained is very good. However, for a PID controller tuned based on another point of the ultimate surface, $K_u = [1.27 \ -0.25]$ and $\omega_u = 0.5129 \text{ rad/s}$, the system became unstable, as shown in Fig. 7. These results show that the SISO tuning is too risky, due to the non-existence of a full multivariable analysis to justify the tuning procedure.

B. PID controller obtained with Multivariable Ultimate Point Method

Next, we have found a PID controller that moves the characteristic loci to the point $-0.45 - 0.42j$, which is the equivalent of Tyreus-Luyben point for the SISO Ultimate Point Method. As (23) yields two solutions, Fig. 9 presents the characteristic loci of the system with each one of the controllers inserted.

The outermost characteristic loci of both systems - system controlled by PID 1 and system controlled by PID 2 - cross the circle of unitary radius approximately at the same point. At every point of this circle, the magnitude of the system's response is equal to one and the phase margin is given by the angle between the crossing of the outermost characteristic locus with the circle of unitary radius and the negative real axis. This means that the phase margin of both systems is the same. Table IV presents the controllers' gains and the stability margins and Fig. 8 presents both systems responses for different references. As can be seen in Table IV, the phase margins are almost the same for the two controllers - close to 33° - but PID 2 presents a worse gain margin. The effect of this fact can be seen in the step responses of the systems presented in Fig. 8, in which $y_2(t)$ of the system controlled by PID 2 presents a significant overshoot for reference $r = [0 \ 1]'$. Fig. 8 presents the step responses of the system with the controllers obtained with $\hat{G}_1(j\omega_u)$.

The performance of the system can be improved through

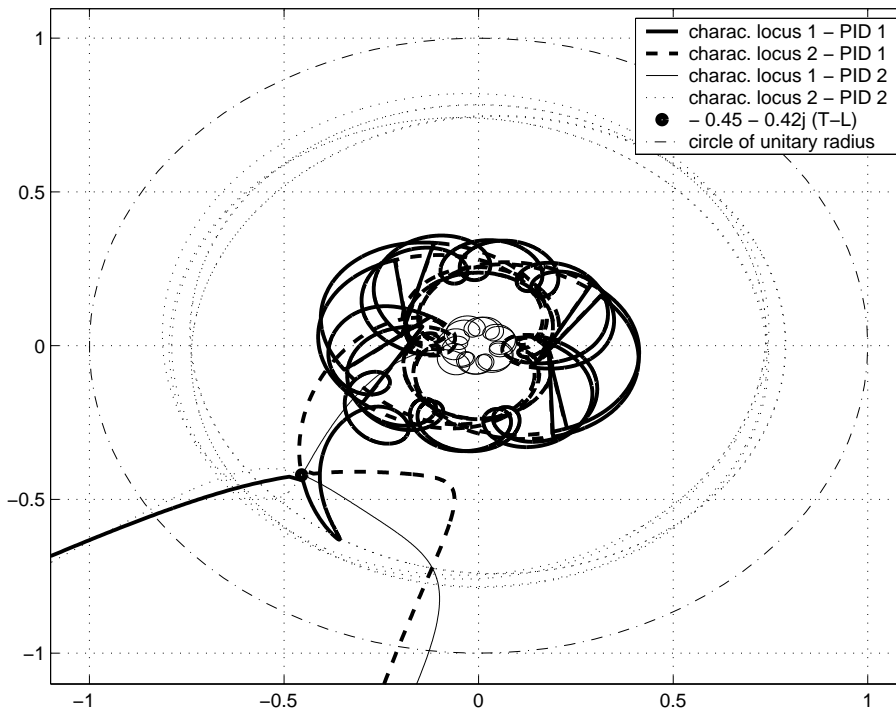


Fig. 9. Characteristic loci of the distillation column with both PID controllers, designed with the proposed method.

TABLE IV
PID CONTROLLERS' GAINS OF THE SYSTEMS PRESENTED IN FIG. 8.

Controller	k_{p1}	k_{i1}	k_{d1}	k_{p2}	k_{i2}	k_{d2}	G_M	Φ_M
PID 1	0.5823	0.2247	0.3773	-0.0860	-0.0135	-0.1374	2.57I	33.25°
PID 2	0.1435	0.0623	0.0826	-0.3123	-0.0434	-0.5617	1.23I	32.59°

TABLE V
PID CONTROLLERS' GAINS THAT DISLOCATE THE ULTIMATE POINT TO $-0.2 - 0.3j$.

\hat{G}	Controller	k_{p1}	k_{i1}	k_{d1}	k_{p2}	k_{i2}	k_{d2}	G_M	Φ_M
1	PID 1	0.3663	0.1102	0.3044	-0.0432	-0.0051	-0.0912	3.8760I	42.50°
1	PID 2	0.0929	0.0311	0.0693	-0.1510	-0.0155	-0.3671	1.8975I	45.78°
2	PID 1	0.3803	0.1149	0.3147	-0.0416	-0.0045	-0.0952	3.2457I	43.05°
2	PID 2	0.0968	0.0315	0.0742	-0.1493	-0.0147	-0.3796	1.8325I	47.09°

the choice of another point in the complex plane, more conservative than Ziegler-Nichols and Tyreus-Luyben ones. Several controllers were designed for different points in the complex plane (Λ 's) and Figs. 11 and 12 present the step responses of the system with PID 1 and 2, respectively, that dislocate the ultimate point to $-0.2 - 0.3j$. Table V presents the controllers' gains and stability margins. In this case, we present the controllers obtained with $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$, which correspond to two different ultimate points. Different controllers were designed, with different ultimate points, and all these controllers yield similar results.

The system performances corresponding to $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$ are not visually distinguishable, reinforcing the fact that the choice of the "right" ultimate point loses importance when the MIMO ultimate point method is applied. This is also verified by the controllers' gains obtained for different ultimate points, which are very similar.

Also, as the point in the complex plane chosen is more conservative than the one used before, there is a significant

increase in the stability margins; step responses just validate that.

C. PI controller obtained with Multivariable Ultimate Point Method for one characteristic locus

Let us present a simpler alternative to the previous choice of dislocation for the ultimate point. The stability limit is usually reached when only one characteristic locus crosses the point $-1 + j0$. If we assume that the other characteristic loci are far enough from -1 , then the insertion of a controller in the loop, however changing the size and shape of them, should not make them involve -1 . In this case, we may concern ourselves only with the "critical" characteristic locus, that is, the one that crosses -1 , and move only this one to a specified location in the complex plane [13].

We apply this idea to the case study to find a controller that moves the ultimate point of the characteristic locus that crosses the point $-1 + j0$ to $-0.40 + 0.08j$, which is the equivalent of Ziegler-Nichols point for SISO Ultimate Point Method using

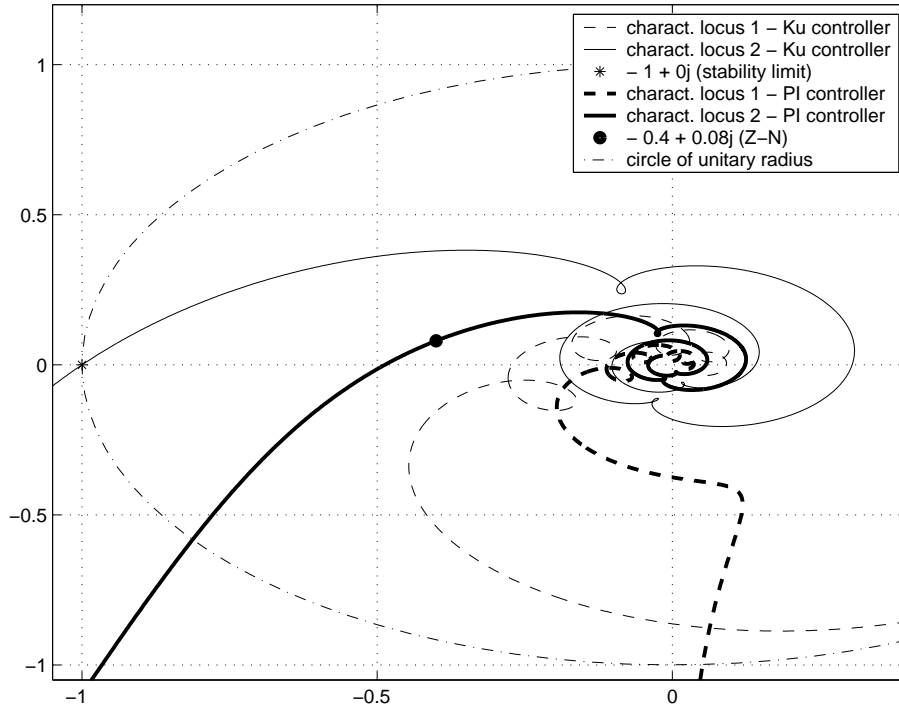


Fig. 10. Characteristic loci of the distillation column with the ultimate gains and with PI controller designed with the proposed method for one characteristic locus.

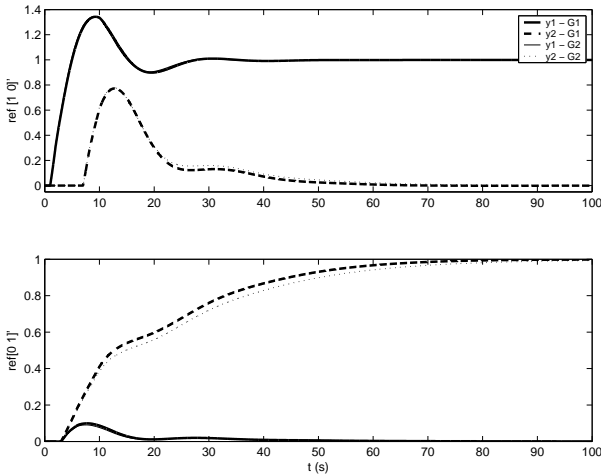


Fig. 11. Step response of the system with PID 1 controller that dislocates the ultimate point to $-0.2 - 0.3j$.

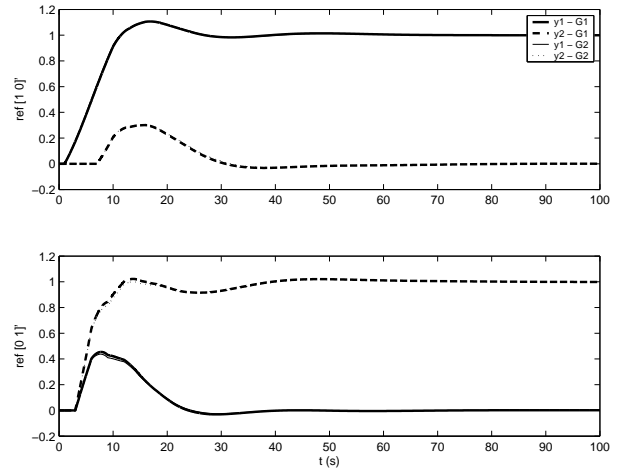


Fig. 12. Step response of the system with PID 2 controller that dislocates the ultimate point to $-0.2 - 0.3j$.

PI control. Fig. 10 presents the characteristic loci of the system with the ultimate gain and with the PI controller inserted; Fig. 13 presents the step response of the system with the controller obtained using $\hat{G}_1(j\omega_u)$; Table VI presents the controller's gains.

The step responses shown in the results present good performance: the overshoot is less than 10% and the settling time is around 40s. As can be seen in Fig. 10, the outermost characteristic locus from the system with PI crosses the circle of unitary radius far from the point $-1 + j0$, and the stability margins can be easily calculated from the Nyquist diagram

presented in Fig. 10:

$$\Phi_M = 38.53^\circ \quad G_M = 2.08I.$$

TABLE VI
PI CONTROLLER'S GAINS OF THE SYSTEM PRESENTED IN FIG. 13.

k_{p1}	k_{i1}	k_{p2}	k_{i2}
0.1840	0.0151	-0.1214	-0.0100

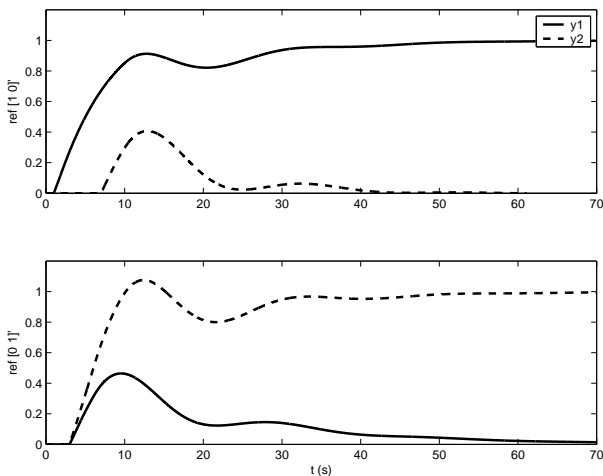


Fig. 13. Step response of the system with PI controller obtained through the Multivariable Ultimate Point Method based on Ziegler-Nichols point.

VII. CASE STUDY - PROCESS II

In this section, we present some results obtained for another process, whose matrix transfer function is presented in (38). This process was also used in [17], [18], and is considered to have strong coupling.

$$G(s) = \begin{bmatrix} \frac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \frac{-1}{(0.1s+1)(0.2s+1)^2} \\ \frac{1}{(0.1s+1)(0.2s+1)^2} & \frac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{bmatrix} \quad (38)$$

First of all, we have estimated $\hat{G}(j\omega_u)$ for different ultimate frequencies through DRF tests. Two of them are presented in Table VII.

The controller design for Wood and Berry distillation column, in the previous section, presented better results with the application of the method that tunes a PID which dislocates both characteristic loci to a desired point. This is mainly due to the fact that, regardless of the ultimate point used, similar performances are obtained, i.e., similar stability margins. The choice of the desired point in the complex plane is what determines how large these margins are.

Another important point to study is the influence of the error in the estimation of $\hat{G}(j\omega_u)$. Fig. 14 presents the Nyquist diagrams of the systems controlled by PID 1, obtained with $G(j\omega_u)$ directly from the transfer function (38) and with $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$, which presents a larger estimation error. The design aims to dislocate the ultimate point to $-0.45 - 0.42j$. Although the tuning that uses $\hat{G}_1(j\omega_u)$ does not reach exactly the desired point, the resulting stability margins are indistinguishable from the ones obtained with the real $G(j\omega_u)$. The tuning obtained for $\hat{G}_2(j\omega_u)$ places the ultimate point even farther from the desired location. In this case, the gain margin has been unchanged, but the phase margin has been decreased significantly for PID 1.

Besides, Fig. 15 presents the Nyquist diagrams for the same design, but in this case we compare the design results when PID 2 is applied. It can be seen that the estimation error has a large influence in the tuning method, which is unable to reach the desired point in the complex plane but, by the other hand, the stability margins have not been changed

significantly. Table VIII presents the controller gains, obtained with $G(j\omega_u)$, $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$, as well as the stability margins obtained with each controller. It can be seen that PID 2 seems to be the best controller whatever estimation of $G(j\omega_u)$ is used for this process. Thus, if we choose PID 2, there is no significant influence of the estimation error in the performance of the system. This can also be seen in Figs. 16 and 17, which present the system response when the controller PID 2 is applied and the estimations $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$, respectively.

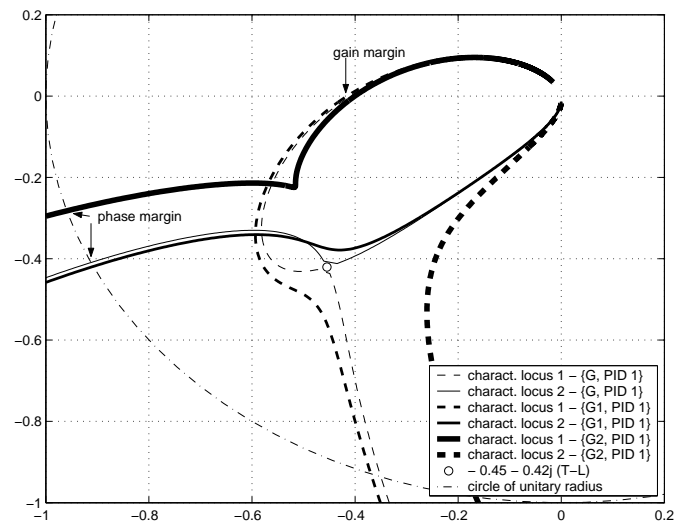


Fig. 14. Characteristic loci of Process II with PID 1, when using the real $G(j\omega_u)$ and the estimations $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$.

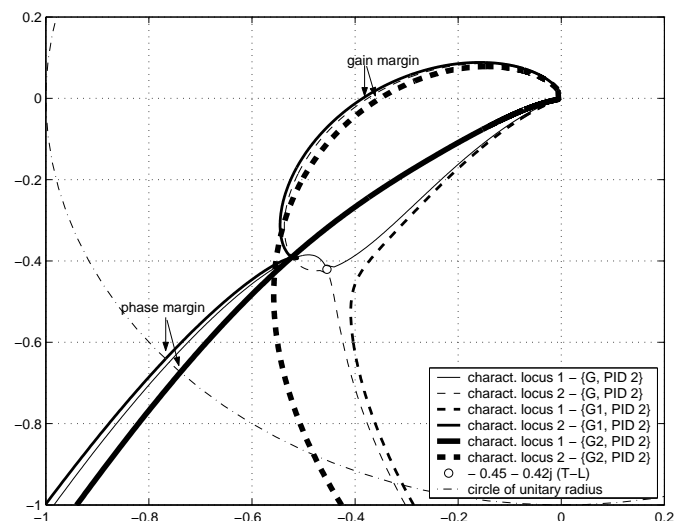


Fig. 15. Characteristic loci of Process II with PID 2, when using the real $G(j\omega_u)$ and the estimations $\hat{G}_1(j\omega_u)$ and $\hat{G}_2(j\omega_u)$.

VIII. CONCLUSIONS

An extension of the Ultimate Point Method for multivariable systems has been proposed, which consists in dislocating the ultimate point to a given location on the complex plane. Different locations on the complex plane can be chosen. In the SISO

TABLE VII
 $\hat{G}(j\omega_u)$ FOR TWO DIFFERENT FREQUENCIES OF (38).

experiment	$ u_2 / u_1 $	ω_u	$\hat{G}(\omega_u)$	ΔG	e (%)
1 2	4.0947 0.7544	4.4366 4.4211	$\hat{G}_1 = \begin{bmatrix} -0.1520 - 0.1743j & 0.1502 + 0.4890j \\ -0.1531 - 0.4845j & -0.5012 - 0.0631j \end{bmatrix}$	0.7	0.7
3 4	0.6250 47.1515	4.4927 4.6776	$\hat{G}_2 = \begin{bmatrix} -0.1736 - 0.1729j & 0.1758 + 0.4491j \\ -0.1460 - 0.4981j & -0.4537 - 0.0142j \end{bmatrix}$	8.4	8.3

TABLE VIII
 PID CONTROLLERS' GAINS FOR PROCESS II THAT DISLOCATES THE ULTIMATE POINT TO $-0.45 - 0.42j$.

\hat{G}	Controller	k_{p1}	k_{i1}	k_{d1}	k_{p2}	k_{i2}	k_{d2}	G_M	Φ_M
$G(j\omega_u)$	PID 1	5.0803	11.4042	0.5658	0.2066	0.8948	0.0119	2.44I	24.16°
$G(j\omega_u)$	PID 2	0.1156	2.4170	0.0014	1.7771	1.7969	0.4394	2.66I	40.47°
$\hat{G}_1(j\omega_u)$	PID 1	5.1997	11.5929	0.5830	0.2097	0.9036	0.0122	2.41I	24.77°
$\hat{G}_1(j\omega_u)$	PID 2	0.1244	2.4665	0.0016	1.7836	1.7843	0.4457	2.60I	39.85°
$\hat{G}_2(j\omega_u)$	PID 1	4.9476	10.4319	0.5866	0.2372	1.0516	0.0134	2.50I	16.40°
$\hat{G}_2(j\omega_u)$	PID 2	0.1033	2.4329	0.0011	1.8223	1.6191	0.5127	2.84I	42.36°

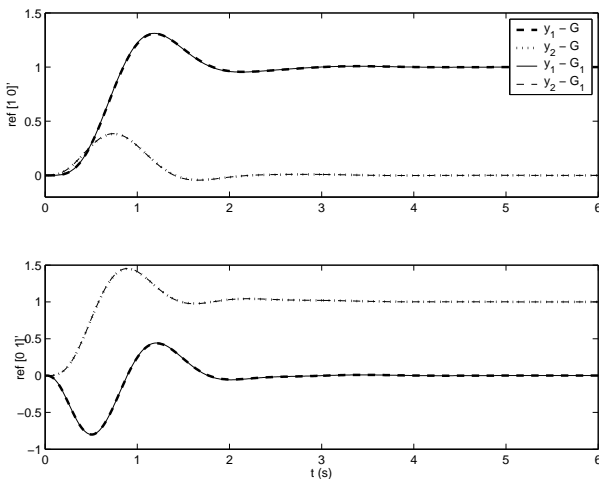


Fig. 16. Step response of Process II with PID 2 obtained with the real $G(j\omega_u)$ and $\hat{G}_1(j\omega_u)$.

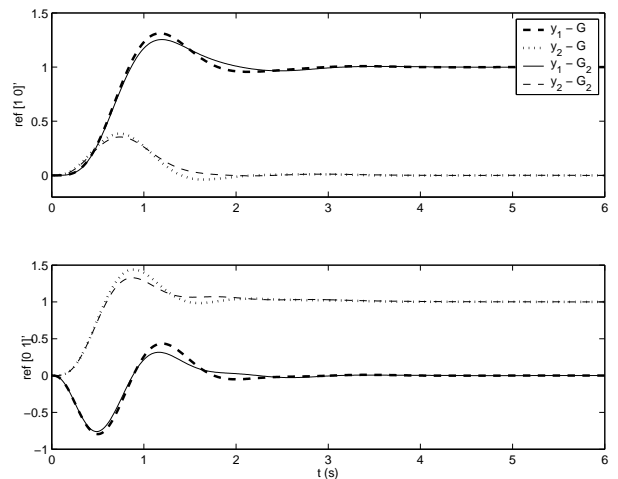


Fig. 17. Step response of Process II with PID 2 obtained with the real $G(j\omega_u)$ and $\hat{G}_2(j\omega_u)$.

case, this chosen location is related to the stability margins of the closed-loop system, provided that some assumptions are satisfied by the frequency response of the process. The same assumptions, when made for the characteristic loci of a MIMO process, will imply the same properties of stability margins. The method proposed inherits the virtues that has made the SISO ultimate point method very successful, but also its shortcomings. Although we have applied this method successfully to a number of relevant examples, it is not clear at this point for which classes of MIMO systems these assumptions are satisfied and how generic these properties are. This is an important area of future research.

On the other hand, previously reported tuning methods for MIMO systems based on the MIMO ultimate quantities are strongly dependent of the particular ultimate point identified. These designs can fail if a “bad” ultimate point is identified, and there are no guidelines to prevent this situation. Our method does not suffer from this drawback, as the use of

different ultimate points for the design lead to quite similar performance, as can be seen by the stability margins presented in Tables V and VIII.

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