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Data-driven Correlation Approach Applied to Load Disturbance Rejection in a Thermal Process*

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Abstract— **From a practical point of view, adjusting the controller without having to identify the process model has many advantages, for example, when the process is simple but changes a lot during the operation. In this case, there are many direct data-driven methods in the literature which may be employed to adjust a monovariable controller aiming at reference tracking. However, when the control objective is disturbance rejection or regulation, the designer is left with too few choices. The aim of this paper is to provide one new option and show how it can be applied to those control objectives.**

I. Introduction

The necessity for retuning a controller is present in many practical situations where the process may deviate from its nominal behaviour along the time. Thermal processes are well known instances of such group. At the same time, re-identifying the process and recalculating the controller for small frequent deviations is tedious and, perhaps, even a costly task. On the other hand, data driven direct methods may be employed to automatically tune the controller's parameters direct from the data [1]. Besides, data driven methods may perform at least as good as model based methods when the controller available is of full order. But data driven methods are proven to give better estimates for the controller's parameters when it is underparameterized, i.e. with reduced order [2], [3].

A reasonable number of direct data driven methods are already proposed in the literature. Examples include: the Virtual Reference Feedback Tuning (VRFT) [4], the Iterative Feedback Tuning (IFT) [5], the Optimal Controller Identification (OCI) [3], and the Noniterative Correlation-based Tuning (NCbT) [6]. All these methods are very good at solving the reference tracking problem. On the other hand, when the designer faces load disturbance rejection problems, there are very few options left. In fact, the only other method for this class of problem, that we know of, is the Virtual Reference Feedback Tuning (VDFT) [7], a variation of the VRFT method for load disturbance response tuning.

With all that in mind, the main objective of this paper is provide another data driven direct method capable of solving the load disturbance problem. The proposed method may be seen as a modification of the NCbT to deal with the load disturbance. This method was first proposed by [8] for monovariable systems. In that work, the controller is tuned from open loop experimental data or closed loop data with a simulated disturbance, and the method was only validated through simulations. That method was later extended to multivariable systems by [9], this time allowing the experiment to excite the reference inputs, but again validated only through simulations. Now, this paper presents the monovariable case with excitation through the reference input and validates the solution with a real thermal process controlled by a commercially available controller.

The remaining of this paper is organized as follows: Section II presents some common definitions regarding the system, controller, control objectives, experiment and noise; then Section III presents the error variable proposed and data driven correlation approach and the solution to the correct controller's parameters estimation problem; after that, Section IV presents the filters for data pre-filtering when the controller is underparameterized or the reference model cannot be perfectly achieved. Finally, Section V presents a case study employing the proposed method to tune the parameters of a commercial controller used to regulate a thermal process. Then, Section VI draws some conclusions from the results.

II. Preliminaries

When in open loop, the monovariable process to be controlled is described by the following classic input-output relationship:

$$y(t) = G(q)u(t) + v(t), \tag{1}$$

where $G(q)$ is the linear time-invariant (LTI) discrete time process model, a rational function of the backward time shift operator, q^{-1}; while $u(t)$ is the control input; and $y(t)$ is the measured output. The latter may be contaminated with a zero-mean, not necessarily white, additive measurement noise $v(t)$.

On the other hand, when in closed loop, the control input is generated by a monovariable LTI parameterized controller $C(q, \boldsymbol{\rho})$, and may be affected by an additive load disturbance $d(t)$, such that

$$u(t, \boldsymbol{\rho}) = C(q, \boldsymbol{\rho})[r(t) - y(t, \boldsymbol{\rho})] + d(t), \tag{2}$$

where $y(t, \boldsymbol{\rho})$ is the measured closed loop output. Also, $\boldsymbol{\rho} \in \mathbb{R}^p$ is the controller's parameter vector.

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Combining (1) and (2) results in the following closed loop equations:

$$y(t, \boldsymbol{\rho}) = T(q, \boldsymbol{\rho})r(t) + S(q, \boldsymbol{\rho})v(t) + Q(q, \boldsymbol{\rho})d(t) \quad (3)$$
$$u(t, \boldsymbol{\rho}) = C(q, \boldsymbol{\rho})S(q, \boldsymbol{\rho})[r(t) - v(t)] + S(q, \boldsymbol{\rho})d(t), \quad (4)$$

where $S(q, \boldsymbol{\rho}) = [1+G(q)C(q, \boldsymbol{\rho})]^{-1}$ is the closed loop sensitivity function, $T(q, \boldsymbol{\rho}) = 1-S(q, \boldsymbol{\rho})$ is the complementary sensitivity function, and $Q(q, \boldsymbol{\rho}) = G(q)S(q, \boldsymbol{\rho})$ is the load disturbance sensitivity function.

This paper presents a data-driven method for adjusting the controller's parameters in order to shape the load disturbance response, such that the following disturbance response cost is minimized:

$$J^{\mathrm{DR}}(\rho) = \|[Q_{\mathrm{d}}(q) - Q(q, \boldsymbol{\rho}))]d(t)\|_2^2, \quad (5)$$

where $Q_{\mathrm{d}}(q)$, known in the literature as the *reference model*, represents the desired closed loop behaviour. The *ideal controller* is the one that forces the closed loop behaviour to match the reference model exactly.

Observe that, for any controller,

$$Q^{-1}(q) = G^{-1}(q) + C(q), \quad (6)$$

from the very definition of the load disturbance sensitivity. Particularly, the ideal controller is given by

$$C^{\star}(q) = Q_{\mathrm{d}}^{-1}(q) - G^{-1}(q), \quad (7)$$

and could be easily calculated, if the process model were available. The ideal controller may be unstable or even non-causal; therefore, exact matching is possible only if the following condition holds.

Condition 1 (matching condition)**.** *The ideal controller may be represented with the controller structure available:*

$$\exists \boldsymbol{\rho}^{\star} \in \mathbb{R}^p \,|\, C(q, \boldsymbol{\rho}^{\star}) = C^{\star}(q). \quad (8)$$

In this case $\boldsymbol{\rho}^{\star}$ is called the ideal parameter vector. *Conversely, this means that the reference model is well chosen and may be achieved with the ideal parameter vector:*

$$Q_{\mathrm{d}}(q) = Q(q, \boldsymbol{\rho}^{\star}). \quad (9)$$

Since this is a data-driven method, instead of using the process model in (7) to minimize (5), a batch of N input-output data samples is collected during an experiment, and those data carry implicit information about the process. The data collected are employed later in the data-driven optimization. Should open loop experimentation be possible, the data collected comprise the following dataset:

$$\mathcal{Z}_u^N = \{u(1), \ldots, u(N), y(1), \ldots, y(N)\}. \quad (10)$$

However, if only a closed loop experiment is possible, but a reference change is permitted, the following dataset is collected:

$$\mathcal{Z}_r^N = \{r(1), \ldots, r(N), u(1), \ldots, u(N), y(1), \ldots, y(N)\}. \quad (11)$$

Finally, if none of the above is possible, a disturbance in the process input may be simulated by adding an offset $d(t)$ to the controller's output, producing the following dataset:

$$\mathcal{Z}_d^N = \{d(1), \ldots, d(N), u(1), \ldots, u(N), y(1), \ldots, y(N)\}. \quad (12)$$

Still regarding the experiment, along this paper the following assumption is employed w.r.t. the noise.

Assumption 2 (the noise)**.** *The measurement noise affecting the system's output is a quasi-stationary process uncorrelated with the other input exciting the system during the experiment. Which means that, during an experiment exciting some input $x(t)$,*

$$\lim_{\mathrm{N}\to\infty} \frac{1}{\mathrm{N}} \sum_{\mathrm{t}=1}^{\mathrm{N}} \mathrm{E}\, x(t)v(t-\tau) = 0 \qquad \forall\tau. \quad (13)$$

This is a common assumption in system identification and is paramount for the correlation approach. Also, only one of the inputs is excited during the closed loop experiment.

III. Correlation approach

Although the final goal is to minimize the cost (5), that function depends on the process model, which is supposed to be unavailable. Therefore, instead of optimizing (5) directly, the correlation approach seeks for the minimizer of the correlation between certain error variable and the experiment input. For this approach to work, the error variable must be:

- **C1** obtainable without the process model. This stems from the fact that the process model is unavailable; therefore, the error variable must depend only on the dataset and on the other known problem pieces: the reference model and the controller structure;
- **C2** a function of the controller's parameters. Therefore, the parameters are the optimization variables;
- **C3** uncorrelated with the experiment input for the ideal parameters. This ensures that the ideal parameters are the global minimum of the correlation.

With that in mind the following error variable is proposed:

$$\varepsilon(t, \boldsymbol{\rho}) = Q_{\mathrm{d}}(q)u(t) - y(t) + C(q, \boldsymbol{\rho})Q_{\mathrm{d}}(q)y(t) \quad (14)$$

Clearly, (14) has the characteristics **C1** and **C2**. For the characteristic **C3**, first observe the following lemma:

Lemma 3. *The error* (14) *from open loop data is*

$$\varepsilon(t, \boldsymbol{\rho}) = S^{-1}(q, \boldsymbol{\rho})[Q_{\mathrm{d}}(q) - Q(q, \boldsymbol{\rho})]u(t) - [1 - C(q, \boldsymbol{\rho})Q_{\mathrm{d}}(q)]v(t), \quad (15)$$

calculated replacing (1) *in* (14) *and grouping the terms due to each external input together.*

Proof. The lemma follows from

$$\varepsilon(t, \rho) = [Q_{\mathrm{d}} + CGQ_{\mathrm{d}} - G]u - [1 - CQ_{\mathrm{d}}]v \quad (16)$$
$$= [S^{-1}Q_{\mathrm{d}} - G]u - [1 - CQ_{\mathrm{d}}]v \quad (17)$$
$$= S^{-1}[Q_{\mathrm{d}} - Q]u - [1 - CQ_{\mathrm{d}}]v, \quad (18)$$

where t, q, and $\boldsymbol{\rho}$ are omitted for brevity. □

Lemma 4. *The error* (14) *from closed loop data obtained with the initial feedback controller $C_0(q)$ is*

$$\begin{aligned}\varepsilon(t,\boldsymbol{\rho}) = & Q^{-1}(q,\boldsymbol{\rho})[Q_\mathrm{d}(q) - Q(q,\boldsymbol{\rho})]T_0(q)r(t) \\ & + Q^{-1}(q,\boldsymbol{\rho})[Q_\mathrm{d}(q) - Q(q,\boldsymbol{\rho})]Q_0(q)d(t) \\ & - Q_\mathrm{d}(q)[C_0(q) - C(q,\boldsymbol{\rho}) + Q_\mathrm{d}^{-1}(q)]S_0(q)v(t),\end{aligned} \tag{19}$$

calculated replacing (3) *and* (4) *in* (14) *and performing some algebraic manipulations. $S_0(q)$, $T_0(q)$, and $Q_0(q)$ are the sensitivities during the experiment.*

Proof. The lemma follows from

$$\begin{aligned}\varepsilon(t,\rho) = & Q_\mathrm{d}C_0S_0r - Q_\mathrm{d}C_0S_0v + Q_\mathrm{d}S_0d \\ & + CQ_\mathrm{d}T_0r + CQ_\mathrm{d}S_0v + CQ_\mathrm{d}Q_0d \\ & - T_0r - S_0v - Q_0d & (20) \\ = & [Q_\mathrm{d}G^{-1} + CQ_\mathrm{d} - 1][T_0r + Q_0d] \\ & - Q_\mathrm{d}[C_0 - C + Q_\mathrm{d}^{-1}]S_0v & (21) \\ = & Q_\mathrm{d}[G^{-1} + C - Q_\mathrm{d}^{-1}][T_0r + Q_0d] \\ & - Q_\mathrm{d}[C_0 - C + Q_\mathrm{d}^{-1}]S_0v & (22) \\ = & Q_\mathrm{d}[Q^{-1} - Q_\mathrm{d}^{-1}][T_0r + Q_0d] \\ & - Q_\mathrm{d}[C_0 - C + Q_\mathrm{d}^{-1}]S_0v & (23) \\ = & Q^{-1}[Q_\mathrm{d} - Q][T_0r + Q_0d] \\ & - Q_\mathrm{d}[C_0 - C + Q_\mathrm{d}^{-1}]S_0v, & (24)\end{aligned}$$

where t, q, and $\boldsymbol{\rho}$ are omitted for brevity and the equality in (23) uses (6). □

With the two lemmas above, the characteristic **C3** is guaranteed by the following theorem.

Theorem 5. *Under the noise assumption and if the matching condition is met, the correlation between the error* (14) *and the experiment input approaches zero as the controller's parameter vector approaches the ideal one.*

Proof. First consider only noiseless data. In this case, the error approaches zero as the parameter vector approaches the ideal one. This happens regardless if open loop (15) or closed loop (19) data is being used, because $Q(q,\boldsymbol{\rho})$ approaches $Q_\mathrm{d}(q)$ and there is no term left.

Now, consider only open loop data corrupted by noise. In this case, at the ideal parameter vector only the term from $u(t)$ vanishes and the error (15) becomes

$$\begin{aligned}\varepsilon(t,\boldsymbol{\rho}^\star) &= -[1 - C(q,\boldsymbol{\rho}^\star)Q(q,\boldsymbol{\rho}^\star)]v(t) & (25) \\ &= -[1 - T(q,\boldsymbol{\rho}^\star)]v(t) & (26) \\ &= -S(q,\boldsymbol{\rho}^\star)v(t), & (27)\end{aligned}$$

which is filtered noise, uncorrelated with $u(t)$ by assumption.

Finally, consider only closed loop data corrupted by noise. In this case, at the ideal parameter vector the terms from $r(t)$ and $d(t)$ become zero and the error (19) becomes

$$\begin{aligned}\varepsilon(t,\boldsymbol{\rho}^\star) = & -Q(q,\boldsymbol{\rho}^\star)C_0(q)S_0(q)v(t) \\ & - Q(q,\boldsymbol{\rho}^\star)[-C(q,\boldsymbol{\rho}^\star) + Q^{-1}(q,\boldsymbol{\rho}^\star)]S_0(q)v(t) & (28) \\ = & -Q(q,\boldsymbol{\rho}^\star)[C_0(q) - G^{-1}(q)]S_0(q)v(t) & (29) \\ = & -Q(q,\boldsymbol{\rho}^\star)G^{-1}(q)v(t) & (30) \\ = & -S(q,\boldsymbol{\rho}^\star)v(t), & (31)\end{aligned}$$

where (29) uses again (6), while (30) uses the identity:

$$G^{-1}(q) = [G^{-1}(q) + C_0(q)]S_0(q), \tag{32}$$

which can be easily obtained replacing the definition of $Q_0(q)$ in the left side of (6) and multiplying by $S_0(q)$. Again, the error in (31) is simply filtered noise, uncorrelated by assumption with the probing signal during the experiment, either $r(t)$ or $d(t)$. □

Because the error variable (14) possesses the three desired characteristics, it may be employed to estimate the ideal parameters by seeking the minimum of the correlation between the error and the experiment input. But first, a correlation cost function must be defined.

Observe the following approximation of the correlation between the error variable and any signal $x(t)$ calculated for τ shifts:

$$\hat{f}(\boldsymbol{\rho},\tau) = \frac{1}{N}\sum\nolimits_{t=1}^{N} x(t-\tau)\varepsilon(t,\boldsymbol{\rho}), \tag{33}$$

and consider the vector constructed by stacking together $2\ell+1$ such values calculated for $\tau \in [-\ell;\ell]$:

$$\hat{\boldsymbol{f}}(\boldsymbol{\rho}) = \frac{1}{N}\sum\nolimits_{t=1}^{N} \boldsymbol{\zeta}(t)\varepsilon(t,\boldsymbol{\rho}), \tag{34}$$

where $\boldsymbol{\zeta}(t)$ is the *instrument vector*, defined as

$$\boldsymbol{\zeta}^\mathsf{T}(t) = \begin{bmatrix} x(t+\ell) & \cdots & x(t) & \cdots & x(t-\ell) \end{bmatrix}, \tag{35}$$

while ℓ is a design parameter. Then the 2-norm of the vector (34) defines the correlation cost function:

$$J^\mathrm{DC}(\boldsymbol{\rho}) = \frac{1}{N^2}\left\|\sum\nolimits_{t=1}^{N} \boldsymbol{\zeta}(t)\varepsilon(t,\boldsymbol{\rho})\right\|_2^2. \tag{36}$$

Using this disturbance correlation cost function, an estimate for the ideal parameter may be calculated according with the following corollary.

Corollary 6. *Under the noise assumption and if the matching condition is met, an estimate for the ideal controller's parameter is given by the solution of the optimization problem:*

$$\hat{\boldsymbol{\rho}} = \arg\min_{\boldsymbol{\rho}} J^\mathrm{DC}(\boldsymbol{\rho}), \tag{37}$$

constructing the instrument vector with samples from the experiment input.

The problem in (37) may be non-convex, requiring an iterative numerical optimization. However, an analytic solution is possible when the following condition is met.

Condition 7 (linearly parameterized controller)**.** *The controller is linearly parameterized, which means*

$$C(q,\boldsymbol{\rho}) = \boldsymbol{\rho}^{\mathsf{T}}\boldsymbol{\beta}(q), \tag{38}$$

where $\beta(q)$ is a vector of LTI transfer operators describing the controller's parameterization structure.

Under the above condition the error variable (14) is

$$\varepsilon(t,\boldsymbol{\rho}) = \tilde{u}(t) - \boldsymbol{\phi}^{\mathsf{T}}(t)\boldsymbol{\rho}, \tag{39}$$

where

$$\tilde{u}(t) = Q_{\mathrm{d}}(q)u(t) - y(t) \tag{40}$$

$$\boldsymbol{\phi}(t) = -\boldsymbol{\beta}(q)Q_{\mathrm{d}}(q)y(t). \tag{41}$$

Replacing (39) in (36) yields

$$J^{\mathrm{DC}}(\boldsymbol{\rho}) = \frac{1}{N^2}[\boldsymbol{\xi}^{\mathsf{T}}\boldsymbol{\xi} - 2\boldsymbol{\xi}^{\mathsf{T}}\mathbf{X}\boldsymbol{\rho} + \boldsymbol{\rho}^{\mathsf{T}}\mathbf{X}^{\mathsf{T}}\mathbf{X}\boldsymbol{\rho}], \tag{42}$$

with

$$\boldsymbol{\xi} = \sum\nolimits_{t=1}^{N}\boldsymbol{\zeta}(t)\tilde{u}(t), \qquad \mathbf{X} = \sum\nolimits_{t=1}^{N}\boldsymbol{\zeta}(t)\boldsymbol{\phi}^{\mathsf{T}}(t).$$

Observe that (42) is convex and its minimizer may be obtained analytically by

$$\hat{\boldsymbol{\rho}} = [\mathbf{X}^{\mathsf{T}}\mathbf{X}]^{-1}[\mathbf{X}^{\mathsf{T}}\boldsymbol{\xi}]. \tag{43}$$

IV. Pre-filtering

Note that when the matching condition is met, (37) (or (43)) gives a consistent estimate of the ideal parameter vector. However, when the ideal controller cannot be constructed with the available structure, the global minima of (5) and (36) differ. In this case, the error may be filtered by a filter $W(q)$ in order to force the minimum of (36) close to the minimum of (5). The data are collected as usual, then only the data employed to calculate the error variable are pre-filtered by $W(q)$.

Lemma 8. *With the aid of Parseval's theorem, the disturbance response cost* (5) *may be written in the frequency domain as*

$$J^{\mathrm{DR}}(\boldsymbol{\rho}) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|Q_{\mathrm{d}}(e^{j\omega}) - Q(e^{j\omega},\boldsymbol{\rho})\right|^2\Phi_d(\omega)\,\mathrm{d}\omega, \tag{44}$$

where $\Phi_d(\omega)$ is the power spectral density of the disturbance signal to be rejected.

Theorem 9. *(filter for open loop data) When employing open loop data, the filter $W(q)$ that makes the cost* (36) *asymptotically identical to the cost* (5) *is one such that*

$$\left|W(e^{j\omega},\boldsymbol{\rho})\right|^2 = \frac{\left|S(e^{j\omega},\boldsymbol{\rho})\right|^2\Phi_d(\omega)}{\Phi_x^2(\omega)}, \tag{45}$$

where $\Phi_x(\omega)$ is the spectrum of the experiment input $u(t)$.

Proof. Because of (15), writing the cost (36) in the frequency domain for open loop data gives

$$\lim_{\substack{N\to\infty\\ \ell\to\infty}} J^{\mathrm{DC}}(\boldsymbol{\rho}) = \frac{1}{2\pi}\int_{-\pi}^{\pi}|\Phi_{x\varepsilon}(\omega)|^2\,\mathrm{d}\omega \tag{46}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\left|Q_{\mathrm{d}}(e^{j\omega}) - Q(e^{j\omega},\boldsymbol{\rho})\right|^2}{\left|S(e^{j\omega},\boldsymbol{\rho})\right|^2} \times \left|W(e^{j\omega},\boldsymbol{\rho})\right|^2\Phi_x^2(\omega)\,\mathrm{d}\omega, \tag{47}$$

where $\Phi_{x\varepsilon}(\omega)$ is the cross-power spectral density of the experiment input $u(t)$ and the error variable $\varepsilon(t,\boldsymbol{\rho})$. The filter that makes (47) identical to (44) is the one in (45). ☐

Now consider the following conditions.

Condition 10. *The desired behaviour is not too far from what can be achieved, i.e. $Q_{\mathrm{d}}(e^{j\omega}) \approx Q(e^{j\omega},\boldsymbol{\rho}^\star)$.*

Condition 11. *The signal to noise ratio during the experiment is high enough, which means that in open loop $\Phi_y(\omega) \approx \left|G(e^{j\omega})\right|^2\Phi_x(\omega)$, while in closed loop either $\Phi_y(\omega) \approx \left|Q_0(e^{j\omega})\right|^2\Phi_x(\omega)$ or $\Phi_y(\omega) \approx \left|T_0(e^{j\omega})\right|^2\Phi_x(\omega)$.*

Condition 12. *The experiment input is constructed such that its spectrum is similar to the spectrum of the disturbance to be rejected, which means that $\Phi_x(\omega) \approx \Phi_d(\omega)$.*

Observe that Condition 10 depends on a good matching between the reference model and the controller structure, while Conditions 11 and 12 depend on experiment design.

Corollary 13. *If Conditions 10–12 are met, the filter* (45) *may be approximated by*

$$\left|W(e^{j\omega})\right|^2 \approx \frac{\left|Q(e^{j\omega},\boldsymbol{\rho}^\star)\right|^2\Phi_d(\omega)}{\left|G(e^{j\omega})\right|^2\Phi_x^2(\omega)} \tag{48}$$

$$\approx \frac{\left|Q(e^{j\omega},\boldsymbol{\rho}^\star)\right|^2\Phi_d(\omega)}{\Phi_y(\omega)\Phi_x(\omega)} \tag{49}$$

$$\approx \frac{\left|Q_{\mathrm{d}}(e^{j\omega})\right|^2}{\Phi_y(\omega)}, \tag{50}$$

which forces the minimum of (47) *close to the one of* (44).

Regarding data collected during a closed loop experiment exciting the system through the reference input, the next theorem provides the filter that compensates the mismatching between the controller structure and the reference model.

Theorem 14. *(closed loop filter, probing with reference) When employing closed loop data obtained by exciting the reference input, the filter $W(q)$ that makes the cost* (36) *asymptotically identical to the cost* (5) *is one such that*

$$\left|W(e^{j\omega},\boldsymbol{\rho})\right|^2 = \frac{\left|Q(e^{j\omega},\boldsymbol{\rho})\right|^2\Phi_d(\omega)}{\left|T_0(e^{j\omega})\right|^2\Phi_x^2(\omega)}, \tag{51}$$

where $\Phi_x(\omega)$ is the spectrum of the experiment input $r(t)$.

Proof. Because of (19), writing the cost (36) in the frequency domain for closed loop data exciting through the reference input gives

$$\lim_{\substack{N\to\infty\\ \ell\to\infty}} J^{\mathrm{DC}}(\boldsymbol{\rho}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left|Q_{\mathrm{d}}(e^{j\omega}) - Q(e^{j\omega},\boldsymbol{\rho})\right|^2}{\left|Q(e^{j\omega},\boldsymbol{\rho})\right|^2} \times \left|T_0(e^{j\omega})\right|^2 \left|W(e^{j\omega},\boldsymbol{\rho})\right|^2 \Phi_x^2(\omega)\,\mathrm{d}\omega, \quad (52)$$

where $\Phi_{x\varepsilon}(\omega)$ is the cross-power spectral density of the experiment input $r(t)$ and the error variable $\varepsilon(t,\boldsymbol{\rho})$. The filter that makes (52) identical to (44) is the one in (51). □

Corollary 15. *If Conditions 10–12 are met, the filter* (51) *may be approximated by*

$$\left|W(e^{j\omega})\right|^2 \approx \frac{\left|Q(e^{j\omega},\boldsymbol{\rho}^\star)\right|^2 \Phi_d(\omega)}{\left|T_0(e^{j\omega})\right|^2 \Phi_x^2(\omega)} \quad (53)$$

$$\approx \frac{\left|Q(e^{j\omega},\boldsymbol{\rho}^\star)\right|^2 \Phi_d(\omega)}{\Phi_y(\omega)\Phi_x(\omega)} \quad (54)$$

$$\approx \frac{\left|Q_{\mathrm{d}}(e^{j\omega})\right|^2}{\Phi_y(\omega)}, \quad (55)$$

which forces the minimum of (52) *close to the one of* (44).

Finally, regarding data collected during a closed loop experiment exciting the system through the disturbance input, the next theorem provides the filter that compensates the mismatching between the controller structure and the reference model.

Theorem 16. *(closed loop filter, probing with disturbance) When employing closed loop data obtained by exciting the disturbance input, the filter $W(q)$ that makes the cost* (36) *asymptotically identical to the cost* (5) *is one such that*

$$\left|W(e^{j\omega},\boldsymbol{\rho})\right|^2 = \frac{\left|Q(e^{j\omega},\boldsymbol{\rho})\right|^2 \Phi_d(\omega)}{\left|Q_0(e^{j\omega})\right|^2 \Phi_x^2(\omega)}, \quad (56)$$

where $\Phi_x(\omega)$ is the spectrum of the experiment input $d(t)$.

Proof. Because of (19), writing the cost (36) in the frequency domain for closed loop data exciting through the disturbance input gives

$$\lim_{\substack{N\to\infty\\ \ell\to\infty}} J^{\mathrm{DC}}(\boldsymbol{\rho}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left|Q_{\mathrm{d}}(e^{j\omega}) - Q(e^{j\omega},\boldsymbol{\rho})\right|^2}{\left|Q(e^{j\omega},\boldsymbol{\rho})\right|^2} \times \left|Q_0(e^{j\omega})\right|^2 \left|W(e^{j\omega},\boldsymbol{\rho})\right|^2 \Phi_x^2(\omega)\,\mathrm{d}\omega, \quad (57)$$

where $\Phi_{x\varepsilon}(\omega)$ is the cross-power spectral density of the experiment input $d(t)$ and the error variable $\varepsilon(t,\boldsymbol{\rho})$. The filter that makes (57) identical to (44) is in (56). □

Corollary 17. *If Conditions 10–12 are met, the filter* (56) *may be approximated by*

$$\left|W(e^{j\omega})\right|^2 \approx \frac{\left|Q(e^{j\omega},\boldsymbol{\rho}^\star)\right|^2 \Phi_d(\omega)}{\left|Q_0(e^{j\omega})\right|^2 \Phi_x^2(\omega)} \quad (58)$$

$$\approx \frac{\left|Q(e^{j\omega},\boldsymbol{\rho}^\star)\right|^2 \Phi_d(\omega)}{\Phi_y(\omega)\Phi_x(\omega)} \quad (59)$$

$$\approx \frac{\left|Q_{\mathrm{d}}(e^{j\omega})\right|^2}{\Phi_y(\omega)}, \quad (60)$$

which forces the minimum of (57) *close to the one of* (44).

V. Case Study

In this section a commercial PID controller [10] is adjusted in order to evaluate the applicability of the proposed solution. That controller is part of a thermal plant that also comprises a heating element and a thermocouple type K sensor. The heating element is driven by the controller's internal relay which, in turn, is controlled by a PWM signal with cycling period of 0.5 seconds. The whole process may be approximated by a first order plus delay system and the objective of the adjusting is to tune the load disturbance response in a way that step disturbances are rapidly rejected while preventing large deviations from the setpoint.

First, the controller's self tuning function is employed to obtain a preliminary adjusting from where the designer may choose a better reference model. The resulting controller is a full PID controller. It is worth mentioning that the self tuning function is aimed at reference tracking, resulting in poor load disturbance rejection behaviour. After running the self tuning routine, a setpoint of 120 °C is selected and the system is allowed to settle. Then a closed loop experiment is performed by changing the value of the controller output's bias, i.e. injecting a known disturbance, and collecting the process input and output. The sampling period is 1 second and the disturbance employed comprises 4 periods of a square wave, each period with 120 samples (2 minutes) with levels $\pm 2\%$ of the pulse width. The whole experiment takes 8 minutes and the data collected is presented in Figure 1.

Sometimes it is difficult to perform an experiment exciting the disturbance input directly, while exciting the system through the reference input is common practice. Therefore, simulating this situation, the data effectively employed to adjust the controller's parameters is collected during another closed loop experiment where, this time, the reference input (setpoint) is excited with a square wave with levels ±2 °C. The data collected during this new experiment is presented in Figure 2. That data compose the dataset $\mathcal{Z}_r^{480}$ that will be employed during the tuning.

The desired closed loop response is represented by the

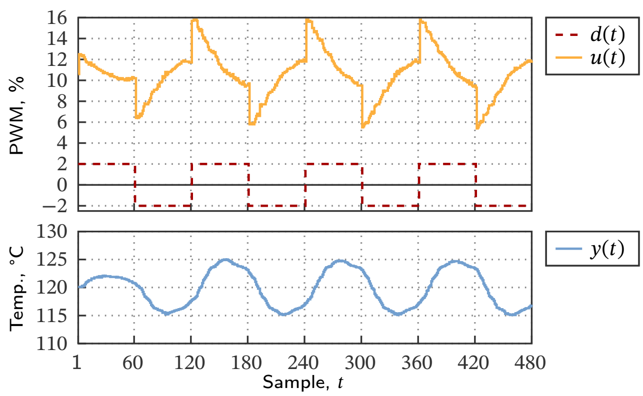

Fig. 1. System's initial response to a square wave disturbance.

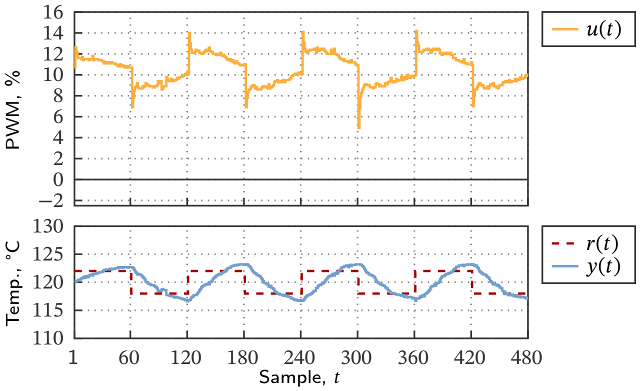

Fig. 2. Data collected with a square wave reference.

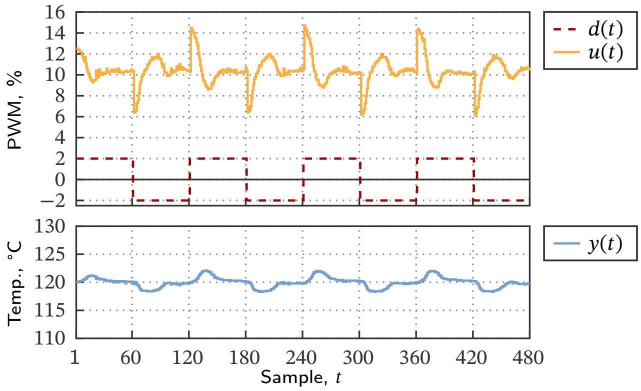

Fig. 3. System's final disturbance response, after the tuning.

following reference model:

$$Q_{\text{d}}(q) = \frac{0.2q^{-1}(1-q^{-1})}{(1-0.9q^{-1})^2}, \tag{61}$$

which rejects step disturbances in less than 60 seconds and presents a peak with less than 0.4 times the disturbance's amplitude in the system's units. The controller structure chosen to be tuned is a PI controller, i.e. the derivative part of the controller is turned off. Therefore, the controller is parameterized as follows:

$$C(q, \boldsymbol{\rho}) = K_p + K_i \frac{1}{1-q^{-1}}, \tag{62}$$

where K_p and K_i are the proportional and integral gains to be estimated.

The controller's optimal parameters are estimated through (43) using the instrumental variable (35) constructed with samples of the reference signal and $\ell = 60$. On the other hand, $\tilde{u}(t)$ and $\boldsymbol{\phi}(t)$ are constructed with data filtered through the following filter:

$$W(q) = Q_{\text{d}}(q)F(q), \tag{63}$$

from (55), where $F(q)$ is a FIR filter whose coefficients are taken from the inverse Fourier transform of $1/Y(\omega)$.

Because the actual controller's parameterization is different from (62), the parameters estimated must be converted to the actual controller's. After adjusting the controller's parameters, another experiment is performed, similar to the first one. The data obtained during that second experiment is presented in Figure 3, representing the final closed loop behaviour. Observe that the system's disturbance response is much faster and has a much lower peak amplitude than the initial response to the same signal presented before in Figure 1.

VI. Conclusion

This paper presented a direct data driven method for adjusting the controller's parameters in order to force the closed loop load disturbance response close to the response of a desired reference model. The proposed method uses the correlation approach from the system identification literature to estimate the optimal parameters regarding the load disturbance response. In order to evaluate the applicability of the proposed solution, a case study is performed on a thermal process controlled by a commercial controller. A batch of data is collected during a closed loop experiment exciting the system through the reference input and the controller's parameters are estimated by the method proposed. The system's final disturbance response is compared with the response obtained with the controller's own self tuning routine. The method proposed is capable of tuning the commercial controller and presents a better disturbance response, indicating its applicability.

References

[1] A. S. Bazanella, L. Campestrini, and D. Eckhard, *Data-Driven Controller Design: The H_2 Approach.* Dordrecht: Springer Science & Business Media, 2011.

[2] S. Formentin, K. van Heusden, and A. Karimi, "A comparison of model-based and data-driven controller tuning," *International Journal of Adaptive Control and Signal Processing*, vol. 28, no. 10, pp. 882–897, 2014.

[3] L. Campestrini, D. Eckhard, A. S. Bazanella, and M. Gevers, "Data-driven model reference control design by prediction error identification," *Journal of the Franklin Institute*, vol. 354, no. 6, pp. 2628–2647, 2017.

[4] M. C. Campi, A. Lecchini, and S. M. Savaresi, "Virtual reference feedback tuning: a direct method for the design of feedback controllers," *Automatica*, vol. 38, no. 8, pp. 1337–1346, 2002.

[5] H. Hjalmarsson, M. Gevers, S. Gunnarsson, and O. Lequin, "Iterative feedback tuning: theory and applications," *IEEE control systems*, vol. 18, no. 4, pp. 26–41, 1998.

[6] K. van Heusden, A. Karimi, and D. Bonvin, "Data-driven model reference control with asymptotically guaranteed stability," *International Journal of Adaptive Control and Signal Processing*, vol. 25, no. 4, pp. 331–351, 2011.

[7] D. Eckhard, L. Campestrini, and E. C. Boeira, "Virtual disturbance feedback tuning," *IFAC Journal of Systems and Control*, vol. 3, pp. 23–29, 2018.

[8] R. W. P. da Silva and D. Eckhard, "Extension of the correlation-based tuning method for load disturbance rejection," in *Proceedings of the 2019 IEEE 58th Conference on Decision and Control (CDC).* Nice, France: IEEE, 2019, pp. 8386–8391.

[9] ——, "Multivariable correlation-based tuning for load disturbance rejection," in *Proceedings of the 21st IFAC World Congress (Virtual).* Berlin, Germany: IFAC, 2020, pp. 4043–4048.

[10] Novus, "Controlador n1200," online, 2021. [Online]. Available: https://www.novus.com.br/downloads/Arquivos/manual_n1200_v20x_j_pt.pdf