Virtual Disturbance Feedback Tuning

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Abstract

This paper presents a data-driven control method formulated for the disturbance rejection problem. Inspired on the Virtual Reference Feedback Tuning method, which is based on a tracking reference model, the proposed methodology, entitled *Virtual Disturbance Feedback Tuning*, is based on a disturbance model. Using only input/output data collected on the process (no process model) and a linearly parameterized controller, the optimal controller parameters are obtained through least squares, resulting in a closed loop system as close as possible to the disturbance model. Experimental results show the efficiency of the proposed methodology.

Keywords: Data-driven control, PID control, Disturbance Rejection, Disturbance Model, VRFT

1. Introduction

Data-driven control methods are techniques that adjust the parameters of controllers directly from input and output data, without using a model of the process. A common theoretical framework for these data-driven methods is given in [1]. Some of these methods are iterative: the parameters of the controller are refined from one iteration to other, using experimental data collected in closed-loop, until the optimal controller is achieved [2, 3, 4]. Others are "one-shot" - that is, they directly estimate the controller's parameters on the basis of one sequence of input-output data [5, 6, 7]. Among the one-shot methods, Virtual Reference Feedback Tuning (VRFT) [5] has been extended to output sensitivity and control effort minimization [8, 9] and widely applied for reference tracking in different applications [10, 11, 12, 13, 14, 15, 16].

Indeed, most of one-shot methods aim to solve a tracking model reference design problem where the objective is to obtain a closed-loop response as close as possible to a desired response defined by a reference model. However, in most industrial applications, disturbance and perturbation occurrences are more frequent than reference changes and the primary objective of the controller is to reject these effects efficiently. Yet, when a tracking model reference approach is used and the desired closed loop response is faster than open loop, the closed loop responds accordingly to reference changes, but the settling time to reject load disturbances is close to the open-loop settling time, which is slower than the desired closed loop response.

A model matching controller design for load disturbance rejection is presented in [17], where a *desired disturbance model* is defined, but it depends on the knowledge of the process model. Considering data-driven approaches, an adapted version of VRFT for continuous-time signals is presented in [18] where the load disturbance problem is addressed as a reference model design by rewriting the reference model as a function of the desired disturbance model and the unknown controller. The articles [19, 20] propose two and three degrees of freedom controllers for disturbance attenuation of Virtual Reference Feedback Tuning, but it is assumed that the disturbance signal can be measured. In [21] a robust controller design is applied through the application of IFT, where the optimization is performed considering the reference tracking term and other terms related to sensitivities, including the one from load disturbance. However, the authors do not define a desired disturbance model.

Once load disturbance rejection is most likely more common than reference tracking in industrial applications, a larger effort on designing data-driven methods to solve such problems is needed. Inspired by the VRFT formulation, this article presents the Virtual Disturbance Feedback Tuning (VDFT) method, which is based on a virtual disturbance signal computed from a desired disturbance model. The proposed method assumes that the disturbance signal cannot be measured and it is able to find the ideal controller for the disturbance rejection problem under ideal conditions. In the practical case, where signals are corrupted with noise and the controller structure is restricted to low order (for instance proportional-integral controllers), instrumental variables and filters are proposed to improve the quality of the controller. Sta-

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tistical properties of the estimate are demonstrated and experimental results show the applicability of the methodology.

2. Model Matching Design

2.1. Preliminaries

Consider a linear time-invariant discrete-time single-input single-output process

$$y(t) = G(q)u(t) + v(t) \tag{1}$$

where $u(t)$ is the process input, $v(t)$ is the output noise, which is a stochastic process with zero mean, $G(q)$ is the process transfer function and q is the time shift operator $qx(t) = x(t+1)$. The process input signal is composed by two terms:

$$u(t) = u_c(t) + d(t) \tag{2}$$

where $d(t)$ is a disturbance signal, $u_c(t)$ is the control signal

$$u_c(t) = C(q,\rho)\left(r(t) - y(t)\right), \tag{3}$$

$r(t)$ is a reference signal and $C(q,\rho)$ is a linear time-invariant controller. The controller is parametrized by $\rho \in \mathbb{R}^n$, and belongs to a user defined controller class defined as $\mathcal{C} = \{C(q,\rho), \rho \in P \subseteq \mathbb{R}^n\}$, where the controllers are linearly parametrized, i.e,

$$C(q,\rho) = \rho^T \bar{C}(q)$$

with $\bar{C}(q)$ being a column vector composed by transfer functions. The block diagram of the system in closed-loop is presented in Fig. 1.

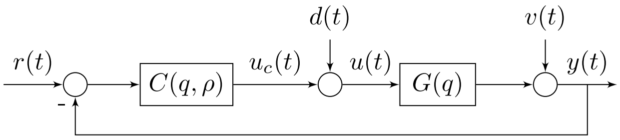

Figure 1: Closed-loop block diagram.

Both $r(t)$ and $d(t)$ are assumed to be quasi-stationary and uncorrelated with the noise, that is $\bar{E}[r(t)v(s)] = 0; \forall t, s$, $\bar{E}[d(t)v(s)] = 0; \forall t, s$ and

$$\bar{E}[f(t)] \triangleq \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E[f(t)]$$

with $E[\cdot]$ denoting expectation [22].

The system (1)-(2)-(3) in closed-loop reads:

$$y(t) = T(q,\rho)r(t) + Q(q,\rho)d(t) + S(q,\rho)v(t) \tag{4}$$

where the terms are defined as: $S(q,\rho) = (1+C(q,\rho)G(q))^{-1}$, $Q(q,\rho) = G(q)S(q,\rho)$ and $T(q,\rho) = C(q,\rho)G(q)S(q,\rho)$.

The role of the control designer is to choose the parameter vector ρ of the controller in order to obtain good performance for the closed-loop system. It is assumed that the user can collect a sufficiently rich batch of process input and output data

$$Z^N = [u(1),\ y(1), \dots, u(N),\ y(N)]$$

and the parameters of the controller are estimated from these data, without the use of a mathematical model of the plant.

Disturbance Model (DM) design consists in specifying a *desired output* for the closed-loop system considering a specified disturbance signal, that is

$$y_{dd}(t) = Q_d(q)d(t),$$

where $Q_d(q)$ is the *Disturbance Model* and then solve an optimization problem where the parameters of the controller are obtained as follows

$$\rho^{DM} = \arg\min\ J^{DM}(\rho),$$

with

$$J^{DM}(\rho) \triangleq \bar{E}[(Q_d(q) - Q(q,\rho))d(t)]^2. \tag{5}$$

An usual choice for the Disturbance Model[2] is to include a zero at 1 to ensure null steady-state gain for constant disturbances and to choose the poles accordingly to the desired settling time.

Equation (5) shows that if the DM *ideal controller*

$$C_d^{DM}(q) = \frac{G(q) - Q_d(q)}{G(q)Q_d(q)}$$

were used in the closed-loop then the objective function (5) would evaluate to zero. However, this controller may not be represented by the controllers class $\mathcal{C}$. When it does, the following assumption holds.

Assumption 1. *Disturbance matching condition*

$$\exists\ \rho_d^{DM} \quad \textit{such that} \quad C(q,\rho_d^{DM}) = C_d^{DM}(q).$$

When Assumption 1 does not hold, the obtained controller $C(q,\rho^{DM})$ is not the DM ideal controller, but it is the best controller that can be used, i.e., it minimizes $J^{DM}(\rho)$ and results in a closed-loop response that is as close as possible to the desired output $y_{dd}(t)$. On the other hand, since the controller $C(q,\rho^{DM})$ is optimal for disturbance rejection, the performance for reference changes is reduced and the closed loop may present overshoot due to an aggressive control action, as it will be seen in the results at the end of the paper.

3. Virtual Disturbance Feedback Tuning

This section describes the main result of the article, which is a direct (non iterative) data-driven approach for the disturbance model design. The method is inspired in

[2]considering minimum phase systems

the Virtual Reference Feedback Tuning [5], and uses a virtual disturbance signal to compute the gains of a linearly parameterized controller aiming to reduce the effect of disturbances on the closed-loop system response.

Consider initially the noise-free and null reference case where $v(t) = 0$ in (1) and $r(t) = 0$ in (3). Through either an open-loop or a closed-loop experiment, input data $u(t)$ and output data $y(t)$ are collected on the process. Given the measured $y(t)$, the *virtual disturbance* signal $\bar{d}(t)$ is defined such that

$$Q_d(q)\bar{d}(t) = y(t),$$

and the virtual control signal is given by

$$\bar{u}_c(t) = u(t) - \bar{d}(t), \tag{6}$$

as shown in Fig. 2.

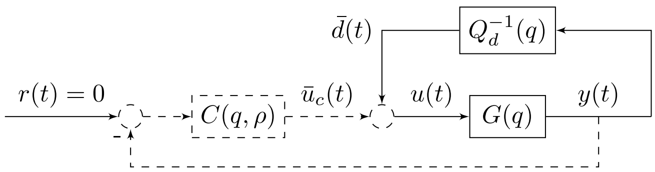

Figure 2: Closed-loop block diagram and the virtual system's signals for the VDFT method.

Even though the plant $G(q)$ is unknown, when it is fed by $u(t)$ (the measured input signal), it generates $y(t)$ as output. So, a "good" controller is one that generates $\bar{u}_c(t)$ when fed by $-y(t)$. Since both signals $\bar{u}_c(t)$ and $-y(t)$ are known, the controller design can be seen as the identification of the dynamical relation between $-y(t)$ and $\bar{u}_c(t)$. As a result of this reasoning, the VDFT method solves the following optimization problem:

$$\rho^{VD} = \arg\min \ J^{VD}(\rho)$$

where

$$J^{VD}(\rho) = \sum_{t=1}^{N} \left\{K(q)\left[\bar{u}_c(t) + C(q, \rho)y(t)\right]\right\}^2, \tag{7}$$

in which $K(q)$ is a filter. Since the controller is linearly parametrized, $J^{VD}(\rho)$ is quadratic and the optimization problem has the following closed solution

$$\rho^{VD} = -\left(\sum_{t=1}^{N} \varphi_K(t)\varphi_K^T(t)\right)^{-1} \sum_{t=1}^{N} \varphi_K(t)\bar{u}_{cK}(t) \tag{8}$$

where

$$\varphi_K(t) = \bar{C}(q)K(q)y(t), \quad \bar{u}_{cK}(t) = K(q)\bar{u}_c(t). \tag{9}$$

In the sequence, estimate properties are presented considering noisy and noiseless cases and satisfaction of Assumption 1.

3.1. Assumption 1 is satisfied: noiseless case

Theorem 1. *When signals are noise-free and Assumption 1 is satisfied, then $C(q, \rho^{VD}) = C_d^{DM}(q)$.*

Proof. Observe that if there is no noise, then $y(t) = G(q)u(t)$. Besides, when Assumption 1 is satisfied $C_d^{DM}(q) = C(q, \rho_d^{DM})$. Now, omitting the dependence on t and q to save space, the criterion can be written as

$$\begin{aligned}
J^{VD}\left(\rho_d^{DM}\right) &= \sum_{t=1}^{N} \left\{K\left[\bar{u}_c + C\left(\rho_d^{DM}\right)y\right]\right\}^2 \\
&= \sum_{t=1}^{N} \left\{K\left[u - Q_d^{-1}y + C_d^{DM}y\right]\right\}^2 \\
&= \sum_{t=1}^{N} \left\{K\left[u - Q_d^{-1}Gu + C_d^{DM}Gu\right]\right\}^2 \\
&= \sum_{t=1}^{N} \left\{K\left[u - \frac{1 + GC_d^{DM}}{G}Gu + C_d^{DM}Gu\right]\right\}^2 \\
&= \sum_{t=1}^{N} \left\{K\left[u - u - GC_d^{DM}u + C_d^{DM}Gu\right]\right\}^2 = 0
\end{aligned}$$

such that ρ_d^{DM} is the minimum of $J^{VD}(\rho)$ for any filter $K(q)$, which implies that $\rho^{VD} = \rho_d^{DM}$. ■

3.2. Assumption 1 is satisfied: noisy case

In the practical case, where signals are corrupted with noise, the controller identification through least squares is biased because the input signal for the identification is corrupted by noise. Aiming to mitigate the bias, an instrumental variable technique is proposed, as it was in the VRFT methodology [5]. It is recommended to use as instrumental variable the output signal of a second experiment, where the same inputs (reference, disturbance and control) signals are applied to the system. In this case, the controller parameter vector is estimated as

$$\rho_{IV}^{VD} = -\left(\sum_{t=1}^{N} \zeta_K(t)\varphi_K^T(t)\right)^{-1} \sum_{t=1}^{N} \zeta_K(t)\bar{u}_{cK}(t) \tag{10}$$

where the instrumental variable is given by

$$\zeta_K(t) = \bar{C}(q)K(q)y'(t) \tag{11}$$

and $y'(t)$ is the output collected on the second experiment. With this IV choice, a statistical property of the estimate can be stated.

Theorem 2. *When signals are noisy, Assumption 1 is satisfied, and the controller parameters are estimated through (10) then asymptotically ($N \to \infty$) $E\left[\rho_{IV}^{VD}\right] = \rho_d^{DM}$.*

Proof. First, observe that $y(t) = G(q)u(t) + v(t)$ and $y'(t) = G(q)u'(t) + v'(t)$ such that the two noises are uncorrelated.

Substituting (9) and (11) in equation (10), and omitting the dependence on t and q to save space, the parameter ρ_{IV}^{VD} can be written as

$$\rho_{IV}^{VD} = -M^{-1}\sum_{t=1}^{N}\left(\bar{C}Ky'\right)\left(K(u - Q_d^{-1}y)\right)$$

where $M = \sum_{t=1}^{N}\left(\bar{C}Ky'\right)\left(\bar{C}Ky\ \right)^T$. Observe that $Q_d^{-1}(q) = (\rho_d^{DM})^T\bar{C}(q) + G^{-1}(q)$ and therefore

$$\begin{aligned}\rho_{IV}^{VD} &= -M^{-1}\sum_{t=1}^{N}\left(\bar{C}Ky'\right)\left(K(-(\rho_d^{DM})^T\bar{C}y - G^{-1}v)\right)\\ &= \rho_d^{DM} + M^{-1}\sum_{t=1}^{N}\left(\bar{C}Ky'\right)KG^{-1}v.\end{aligned}$$

Since $v(t)$ is uncorrelated with $y'(t)$, asymptotically the expected value of the second term is zero and therefore $E\left[\rho_{IV}^{VD}\right] = \rho_d^{DM}$, which concludes the proof. ■

3.3. Assumption 1 is not satisfied

Consider now the case where signals are still noise-free, but Assumption 1 is not satisfied, which means that the chosen controller structure (for instance PID) cannot represent the ideal full order controller $C_d^{DM}(q)$. In this case, the best controller that can run in the loop is $C(q, \rho^{DM})$, which is the controller that minimizes the cost function $J^{DM}(\rho)$. Theorem 1 shows that if Assumption 1 is satisfied then the controller obtained with VDFT is the optimal controller $\rho^{VD} = \rho^{DM}$ but this is not true in general if the assumption is violated. However, the next theorem shows that a specific choice for filter $K(q)$ makes $J^{VD}(\rho)$ and $J^{DM}(\rho)$ exactly the same such that $\rho^{VD} = \rho^{DM}$.

Theorem 3. *Consider the case where $N \to \infty$ and let the filter be given by*

$$\left|K(e^{j\omega})\right|^2 = \left|Q_d(e^{j\omega})Q(e^{j\omega},\rho)\right|^2\frac{\Phi_d(\omega)}{\Phi_y(\omega)},\quad \forall\omega\in[-\pi,\pi]. \tag{12}$$

where $\Phi_d(\omega)$ is the spectrum of the disturbance signal we want to reject and $\Phi_y(\omega)$ is the spectrum of the collected output signal. Then $J^{VD}(\rho) = J^{DM}(\rho)$, and consequently $\rho^{VD} = \rho^{DM}$.

Proof. Applying Parseval's Theorem on (5) and omitting the dependence on ω the criterion becomes

$$\begin{aligned}J^{DM}(\rho) &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{G}{1+C_d^{DM}G} - \frac{G}{1+C(\rho)G}\right|^2\Phi_d(\omega)d\omega\\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{G(1+C(\rho)G) - G(1+C_d^{DM}G)}{(1+C_d^{DM}G)(1+C(\rho)G)}\right|^2\Phi_d(\omega)d\omega\\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|Q_dQ(\rho)\right|^2\left|C(\rho)-C_d^{DM}\right|^2\Phi_d(\omega)d\omega \qquad (13)\end{aligned}$$

Now, substituting (6) into (7) and omitting the dependence on q the virtual disturbance criterion becomes

$$J^{VD}(\rho) = \sum_{t=1}^{N}\left\{K\left[u(t) - \bar{d}(t) + C(\rho)Gu(t)\right]\right\}^2$$

Using the fact that $\bar{d}(t) = (1 + C_d^{DM}G)u(t)$ one gets that

$$\begin{aligned}&J^{VD}(\rho) =\\ &= \sum_{t=1}^{N}\left\{K\left[u(t) - (1+C_d^{DM}G)u(t) + C(\rho)Gu(t)\right]\right\}^2\\ &= \sum_{t=1}^{N}\left\{KG\left(C(\rho) - C_d^{DM}\right)u(t)\right\}^2. \qquad (14)\end{aligned}$$

In the limit case where $N \to \infty$ Parseval's Theorem can be applied to (14), which gives

$$J^{VD}(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi}|KG|^2\left|C(\rho)-C_d^{DM}\right|^2\Phi_u(\omega)d\omega. \tag{15}$$

Comparing (13) and (15) it is clear that if

$$|K|^2 = |Q_dQ(\rho)|^2\frac{\Phi_d(\omega)}{\Phi_y(\omega)}$$

then $J^{VD}(\rho) = J^{DM}(\rho)$, concluding the proof. ■

Notice that $Q(e^{j\omega},\rho)$ is unknown and filter (12) can not be implemented. However, the approximation $|Q(e^{j\omega},\rho)| \approx |Q_d(e^{j\omega})|$ can be made resulting in the implementable filter

$$\left|K(e^{j\omega})\right|^2 \approx \left|Q_d(e^{j\omega})Q_d(e^{j\omega})\right|^2\frac{\Phi_d(\omega)}{\Phi_y(\omega)},\quad \forall\omega\in[-\pi,\pi] \tag{16}$$

which can be used to approximate $J^{VD}(\rho)$ and $J^{DM}(\rho)$. It is important to emphasize that filter (16) is an approximation of the filter described in Theorem 3 and that the cost functions $J^{VD}(\rho)$ and $J^{DM}(\rho)$ will be similar but their minimum will not be exactly the same. Two important remarks on filter (16) need to be considered:

- *Signal spectra estimation:* in order to obtain (16), both disturbance and output spectra are needed. Disturbance spectrum estimation is an easy task if the user knows the class of disturbances to be rejected, as step disturbances, for example. However, since the process model is unknown, the output spectrum need to be estimated from experimental signals;
- *Choice of $Q_d(q)$*: The Disturbance Model has direct influence on the obtained controller. If the desired response is very different from the best performance that can be achieved with the chosen controller structure, then the approximation of the filter is not valid and the obtained controller with VDFT may be very different from $C(q, \rho^{DM})$, which is also very different from $C_d^{DM}(q)$.

4. Practical aspects of the method

When using the VDFT method, there are two important choices the user must make: the class $\mathcal{C}$ of controllers and the desired disturbance model $Q_d(q)$. The desired disturbance model choice directly defines the ideal controller $C_d^{DM}(q)$, while the choice of the controller structure, together with the disturbance model, defines the optimal controller $C(q, \rho^{DM})$. In practice, it is usual that the class of controllers is fixed due to hardware constraints (for instance PID in industrial PLCs) and then it is common that Assumption 1 is violated. The user should keep in mind that since the controller structure is fixed, there are limits on the achievable performance for the closed loop system and that some desired disturbance models may not be reachable. For instance, when a high performance disturbance model is chosen with a low order controller then the ideal controller $C_d^{DM}(q)$ and the optimal controller $C(q, \rho^{DM})$ may be very different. In this case, the performance of the closed loop system with $C(q, \rho^{DM})$ would be much worse than expected[3]. When this happens, the user has two options:

- to choose a different (larger) structure for the controller, such that the difference between the optimal controller and the ideal controller is smaller;
- to specify an "easier" Disturbance Model $Q_d(q)$, such that the controller structure is flexible enough to achieve an acceptable response.

Usually the structure of the controller is chosen *a priori* and then the problem boils down to choosing the disturbance model. When making this choice, the user must keep in mind that there is a limit on the achievable performance, but this limit is unknown since the model is also unknown. For the tracking model reference problem, several authors recommend an iterative procedure for choosing the reference model [1, 23, 24], an approach that can be adapted to the load disturbance problem. The main idea is to choose initially an "easier" disturbance model (with large settling time and high gain), compute the controller with the proposed method and check with an experiment if this disturbance model was achieved. If the output response is close to the desired output, the user may choose a slightly more "difficult" disturbance model (with smaller settling time and smaller gain) which will result in a more aggressive controller. This procedure may be repeated until the point where the current disturbance model cannot be matched by the closed loop system with the chosen controller class.

The proposed procedure is not only useful for determining the desired disturbance model as it is to ensure that approximation used in designing the filter $K(q)$ is valid. Also, if the closed loop tests are done with a known disturbance signal $d(t)$ then

$$\Phi_y(\omega) = |Q(e^{j\omega}, \rho_1)|^2 \Phi_d(\omega),$$

and filter (16) can be written as

$$|K(e^{j\omega})|^2 \approx \frac{\left|Q_d(e^{j\omega})Q_d(e^{j\omega})\right|^2}{|Q(e^{j\omega}, \rho_1)|^2} \frac{\Phi_d(\omega)}{\Phi_d(\omega)} = \frac{\left|Q_d(e^{j\omega})Q_d(e^{j\omega})\right|^2}{|Q(e^{j\omega}, \rho_1)|^2}$$

which does not explicitly depend on signals' spectra. Furthermore, in the case where the disturbance model $Q_d(q)$ is close to the current disturbance $Q(q, \rho_1)$, then the filter can be approximated to $K(q) \approx Q_d(q)$ which is much easier to implement and does not depend on extra procedures.

5. Experimental Results

The proposed methodology was applied to design a controller of a pilot plant, where the goal is to control the liquid level. The same plant was used in [1], where VRFT was used to design the flow control of one tank. The schematic diagram in Fig. 3 describes the main parts of the process, considering a multivariable approach. The whole process is built with of-the-shelf industrial equipments (pumps, valves, sensors and tanks). Tanks 1 and 2 are 70 liters each, while tank 3 is a 250 liters container.

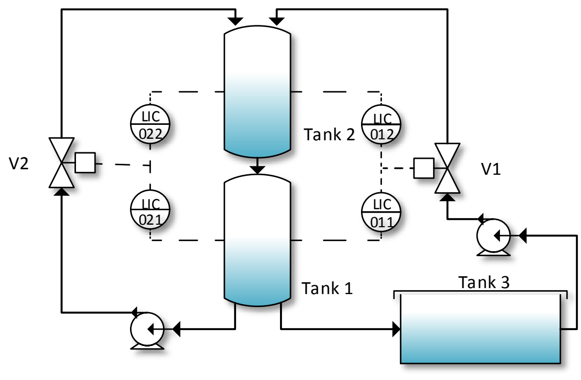

Figure 3: Schematic diagram of the pilot plant.

In this experiment the water is pumped up from Tank 3 to Tank 2 through Valve 1, from Tank 1 to Tank 2 through Valve 2 and back to Tank 3 by gravity. The liquid level of Tank 2 is $y(t)$ and the opening of Valve 1 is the manipulated variable $u(t)$. Also, the opening of Valve 2 is used as disturbance $d(t)$.

The objective of the control system is to control the level of Tank 2 considering changes in the reference and disturbance signals. Virtual Reference Feedback Tuning (VRFT) and Virtual Disturbance Feedback Tuning (VDFT) were applied to design the gains of a PI controller

$$C(q, \rho) = \begin{bmatrix} \rho_1 & \rho_2 \end{bmatrix} \begin{bmatrix} \frac{q}{q-1} \\ \frac{1}{q-1} \end{bmatrix}.$$

[3] Observe that the described problems are inherent to any disturbance model problem, and not only to data-driven approaches.

In order to apply both methods, two open-loop experiments were run to collect data, where the sampling time was $T_s = 1$ s. A step signal was applied to the input of the system, and since the output presents noise the experiment was repeated to use the instrumental variables technique. In both experiments Valve 1 was opened from 40% to 50% while Valve 2 was kept constant at 80%. The output of the first experiment was named $y_1(t)$ while the output of second experiment was named $y_2(t)$. Both signals are presented in Fig. 4.

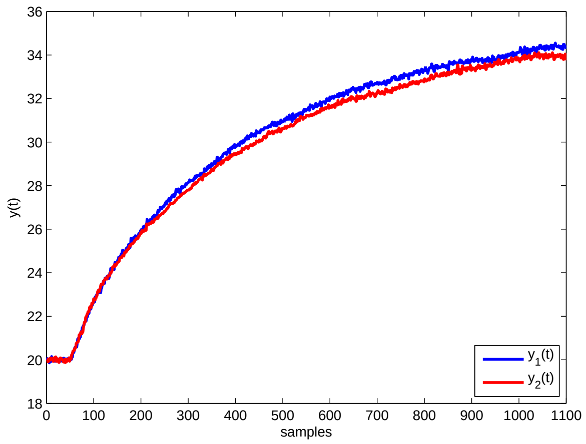

Figure 4: Output signals of the pilot plant, considering two experiments in open-loop, where Valve 1 is opened from 40% to 50%.

The VRFT method was then used to tune the gains of the controller. The choice of the Reference Model was:

$$T_d(q) = \frac{0.015}{q - 0.985} \tag{17}$$

which has a settling time of approximately 260 samples (to achieve 98% of the response) that is faster than the open-loop settling time, which is close to 1000 samples. Two sets of data were used in the VRFT method with instrumental variables with filter $L(q) = T_d(q)(1 - T_d(z))$, and the following controller was obtained:

$$C(q, \hat{\rho}_{IV}^{VR}) = \begin{bmatrix} 2.799 & -2.788 \end{bmatrix} \begin{bmatrix} \frac{q}{q-1} \\ \frac{1}{q-1} \end{bmatrix}. \tag{18}$$

With this controller it is possible to approximate the response of the closed-loop system for disturbances as

$$\hat{Q}(q, \hat{\rho}_{IV}^{VR}) \approx \frac{T_d(q)}{C(q, \hat{\rho}_{IV}^{VR})} = \frac{0.0053591(q-1)}{(q-0.9961)(q-0.985)}. \tag{19}$$

Also, Virtual Disturbance Feedback Tuning (VDFT) was used with the same two sets of data. The choice of the Disturbance Model was:

$$Q_d(q) = \frac{0.005(q-1)}{(q-0.985)(q-0.985)}. \tag{20}$$

This Disturbance Model has a zero at 1 to ensure steady-state disturbance rejection of constant signals and it presents two poles at the same position of the Reference Model. Gain 0.005 was chosen to provide a disturbance response with half amplitude compared to the disturbance response obtained with the VRFT controller (18). The following controller was obtained using (10) and filter $K(q) = Q_d(q)$:

$$C(q, \hat{\rho}_{IV}^{VD}) = \begin{bmatrix} 5.1847 & -5.1395 \end{bmatrix} \begin{bmatrix} \frac{q}{q-1} \\ \frac{1}{q-1} \end{bmatrix}. \tag{21}$$

With this controller it is possible to approximate the response of the closed-loop system for reference changes as

$$\hat{T}(q, \hat{\rho}_{IV}^{VD}) \approx Q_d(q) C(q, \hat{\rho}_{IV}^{VD}) = \frac{0.025924(q-0.9913)}{(q-0.985)^2}. \tag{22}$$

Figure 5 (top) presents the closed-loop responses of the system with both controllers for a step as reference signal from 20 to 25 cm, compared to the reference model $T_d(q)$ output and the estimated closed-loop model response obtained with the VDFT controller and $Q_d(q)$. The figure shows that the response of the system with the VRFT controller is quite similar to the Reference Model. The settling time of the response with the VDFT controller is close to the settling time of VRFT controller but it generates a response with overshoot, as expected (22). Also, the response of the VDFT controller is close to the approximate response computed above. Figure 5 (bottom) presents the corresponding control signals for VRFT and VDFT where it shows that VDFT produces signals with larger magnitude for reference changes.

A similar analysis can be done concerning the disturbance occurrence. Figure 6 (top) presents the closed-loop responses of Tank 1 with both controllers for a step as disturbance signal close to $t = 200$, where Valve 2 was opened from 80% to 100%. Observe that the response of the system with the VDFT controller is quite similar to the Disturbance Model response. The settling time with the VRFT controller is much larger than the settling time of VDFT controller and presents a larger amplitude. Also, the response with VRFT is similar to the approximate disturbance model (19) computed before. Figure 6 (bottom) presents the corresponding control signals and shows that VDFT controller responds a little faster than VRFT controller, but with similar amplitude.

Both designed controllers reacted as expected. The VRFT controller presents a reference response close to the reference model's, but it also presents a slow response for disturbances. On the other hand, VDFT controller presents disturbance response similar to the disturbance model's, but as expected, the reference response presents overshoot and similar settling time compared to the reference model.

6. Conclusions

The article presented a one-shot data-driven control method to be used in a Disturbance Model (DM) design,

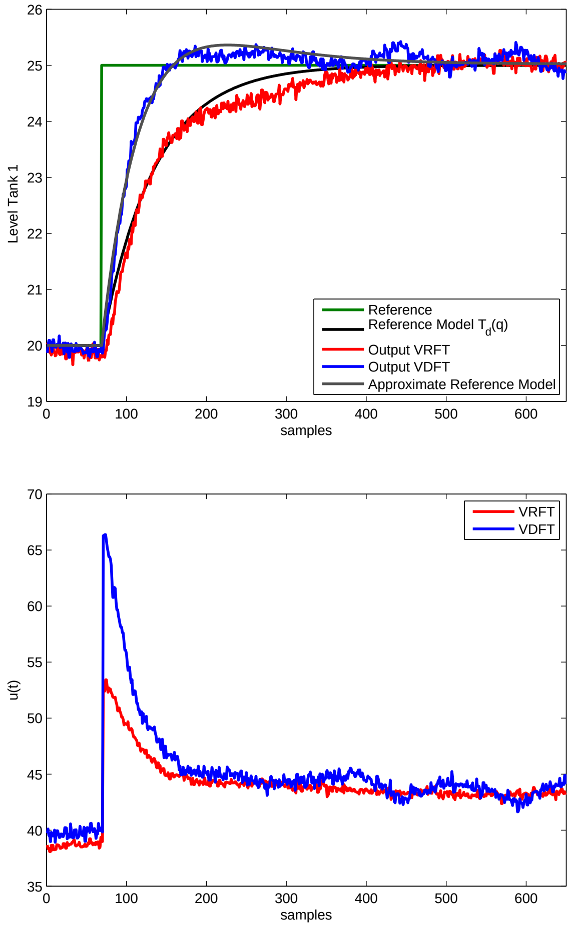

Figure 5: Closed-loop response for reference step from 20 to 25 cm. Top figure: green line is the reference, black line is the output of reference model, red line is the output with VRFT controller, blue line is the output with VDFT controller and grey line is the approximate output computed before the experiment with VDTF controller. Bottom figure: red line is control signal with VRFT controller and blue line is control signal with VDFT controller.

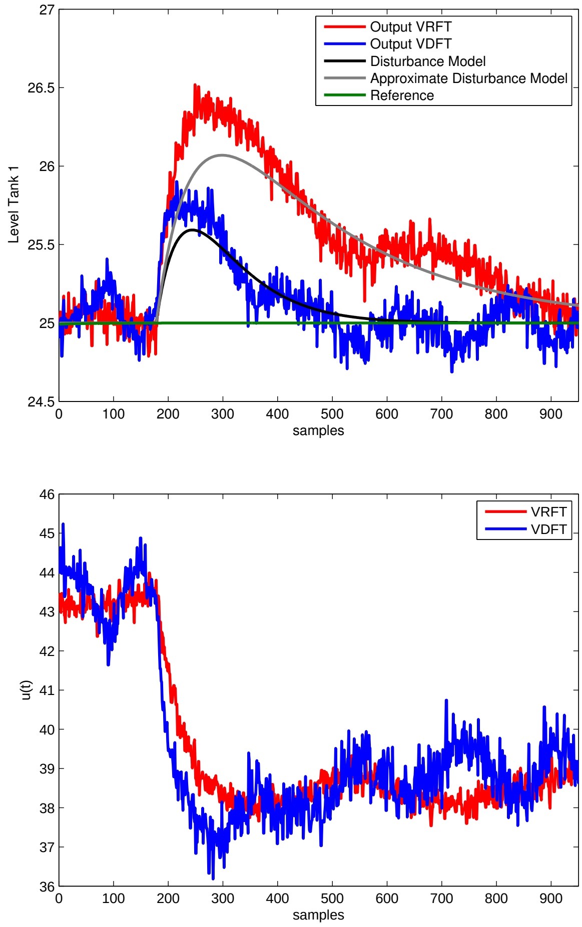

Figure 6: Closed-loop response for disturbance where valve 2 was opened from 80% to 100%. Top figure: green line is the reference, black line is the output of disturbance model, blue line is the output with VDFT controller, red line is the output with VRFT controller and grey line is the approximate output computed before the experiment with VRFT controller. Bottom figure: red line is control signal with VRFT controller and blue line is control signal with VDFT controller.

instead of the commonly used Model Reference approach. Inspired by the Virtual Reference Feedback Tuning method, the proposed approach is based on a *virtual* disturbance signal, obtained from a desired response considering that the disturbance occurs in the input of the process. Thus, the proposed methodology is named "Virtual Disturbance Feedback Tuning" (VDFT). As it happens in the VRFT approach, the controller design can be seen as an identification problem, which is solved through least squares if the controller to be identified is linear in the parameters. When signals are noise-free and the DM ideal controller belongs to the controller class, it is correctly identified with the proposed formulation. When this is not the case, an extra filter and instrumental variables are used to minimize the bias of the estimate. Experimental results have shown the efficiency of the proposed method in a real plant.

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