Modified MIMO Resonant Controller Robust to Period Variation and Parametric Uncertainty

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***Abstract*— In this work a modified Resonant Controller is proposed to deal with the tracking/rejection problem of periodic signals robust to period variations and parametric uncertainties in the plant. The control strategy is based on a resonant structure in series with a notch filter, which will be responsible to improve the robustness to period variation. A robust state feedback controller is designed by solving a linear matrix inequality (LMI) optimization problem guaranteeing the robust stability of the closed loop system. A numerical example is presented to illustrate the method.**

I. Introduction

Applications such as optical disk drives [1], [2], active filters [3] and nanomotion positioning systems [4] have been attracting increasing attention in the literature since they are examples of systems where the signals to be tracked/rejected are periodic or that at least can be considered periodic in a particular timespan. In some of these examples authors must also consider the design of control systems capable to maintain an acceptable level of performance in the presence of parametric uncertainties or period variations.

Robust tracking/rejection under parametric uncertainty can be guaranteed by control techniques based on the Internal Model Principle (IMP) [5] such as repetitive and resonant controllers. Both approaches employ controllers with resonance peaks on the signal fundamental frequency and its harmonics. In the resonant controller [6], the introduction of a second order system in the control loop is required to compensate each harmonic component, leading to a high order controller and an excessive number of tuning parameters. The repetitive approach [7] employs a delay element in a positive feedback loop to achieve infinite gain at the desired fundamental frequency and its harmonics. Despite the simple controller structure, ensuring closed-loop stability is not trivial, especially in the Multiple-Input, Multiple-Output (MIMO) case.

A point in common in IMP based controllers lies in the fact that tracking performance is highly compromised when the period (or frequency) of interest differs from the one considered in the control design [8]. To illustrate this point lets consider the repetitive controller, which is well known to present a severe loss of performance for small period variations such as 5% or 10% of its nominal value [9]. To mitigate this effect we can point out the control strategy known as High Order Repetitive Controller also presented in [9], where multiple memory loops were employed to “enlarge” the high gain region around the nominal frequency and its harmonics. The main drawback of the High Order Repetitive Controller is the so called waterbed effect [10], i.e. attenuation of disturbances around the fundamental frequency and its multiples is improved, while disturbances or noise at intermediary frequencies are amplified. Also, to the best of our knowledge, this control technique can only be applied to MIMO systems by assuming all signals (reference and disturbances) are multiples of the same fundamental frequency.

In this work, the ideas presented in the High Order Repetitive Controller formulation will be extended to resonant controllers by the series interconnection of a resonant structure and a notch filter. In this case, the first is responsible to guarantee perfect tracking at the nominal frequency while the latter improves robustness to frequency variation. Based on a state space formulation, the controller design is addressed by the solution of an optimization problem subject to Linear Matrix Inequality (LMI) constrains that guarantee the robust stability of the closed loop system as well as a desired level of transient performance. In addition, the proposed approach is capable of ensuring tracking/rejection in MIMO systems for references/disturbances with non-multiple period. A numerical example will be considered to illustrate the proposed method.

Notation: The ith component of a vector x is denoted by $x_{(i)}$. $A_{(i)}$ represents the ith row of a matrix $A \in \mathbb{R}^{n\times n}$, $A(i,j)$ is the element located in the ith row and jth column of A and A' means its transpose. $diag\{A_1, A_2\}$ is a block-diagonal matrix obtained from A_1 and A_2, I_m denotes the m-order identity matrix and $0_{m\times n}$ is the $m \times n$ null matrix. $*$ represents symmetric block elements in a matrix.

II. Preliminaries

A. *Open Loop System*

Consider the continuous-time system described by:

$$\begin{array}{rcl} \dot{x}(t) &=& (A+\Delta A(t))x(t) + Bu(t) + B_d d(t) \\ y(t) &=& (C+\Delta C(t))x(t) \\ e(t) &=& r(t) - y(t) \end{array} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the output, $d(t) \in \mathbb{R}^l$ is a vector of disturbances, $r(t) \in \mathbb{R}^p$ is a reference vector to be tracked and $e(t) \in \mathbb{R}^p$ is the output tracking error. Matrices

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$A,\ B,\ B_d$ and C are constant real matrices of appropriate dimensions and suppose the pair (A, B) is controllable.

We also consider that uncertainty matrices $\Delta A(t)$ and $\Delta C(t)$ are defined by [11]:

$$\left[\begin{array}{c} \Delta A(t) \\ \Delta C(t) \end{array}\right] = \left[\begin{array}{c} H_1 \\ H_2 \end{array}\right] \Xi(t) E$$

where $\Xi(t)$ is a time-varying matrix such that $\{\Xi(t) \in \mathbb{R}^{n_u \times n_u}\ ;\ \Xi(t)'\Xi(t) \leq I_{n_u}\}$ and with $H_1,\ H_2$ and E being known real matrices of appropriate dimensions. In this case n_u denotes the number of uncertain parameters.

B. Resonant Controller

A common approach to the tracking (rejection) of periodic references (disturbances) is the so called resonant control which is based on the IMP and adds resonance peaks to the control transfer function at frequencies of interest. According to this principle, it is well known that sinusoidal periodic references with a frequency of ω_0 will be perfectly tracked, just as disturbances with the same frequency will be perfectly rejected, if complex poles with frequency ω_0 are replicated either in the control law or the plant itself. In terms of Resonant Controllers this can e accomplished by the introduction of

$$G_r(s) = \tfrac{\omega_0^2}{s^2+\omega_0^2} \tag{2}$$

in the control loop. Signals that are periodic but not pure sinusoids may still be dealt with based on their Fourier series expansion. In these cases, a pair of complex poles must be added to the controller transfer function for each frequency $\omega_k,\ k = 1, 2, \cdots$ of the expansion. For signals with infinite harmonic content, a usual practice is to consider only the M most significant harmonics at an expense of a residual tracking error which decreases as M increases. Hence, (2) can be rewritten as

$$G_{mr}(s) = \prod_{k=1}^{M} \tfrac{\omega_k^2}{s^2+\omega_k^2}. \tag{3}$$

A problem occurs, however, if the periodic signal is of varying frequency. When this is the case no guarantees can be provided regarding the tracking or rejection of the signal if (3) alone is implemented in the control law. In the following sections we will adapt the resonant structure to increase its robustness with respect to small frequency variations.

III. Notch-Resonant Controller

A. Proposed Controller

Here we propose a modified Resonant Controller that is able to deal with variations of the fundamental reference/disturbance frequency by applying simple loop-shaping techniques. Inspired by the High Order Repetitive Control, the proposed controller "enlarges" the high gain region of the frequency response, adding robustness to small variations around the nominal frequency of the periodic signal. The above methodology is implemented through the addition of a notch filter

$$G_n(s) = \tfrac{s^2+2\zeta_z\omega_0 s+\omega_0^2}{s^2+2\zeta_p\omega_0 s+\omega_0^2} \tag{4}$$

Fig. 1. Resonant controller and the proposed Notch-Resonant Controller for different values of ζ_z and $\zeta_p = 0.001$.

in series with (2), i.e.,

$$G_{nr}(s) = \tfrac{s^2+2\zeta_z\omega_0 s+\omega_0^2}{s^2+2\zeta_p\omega_0 s+\omega_0^2} \cdot \tfrac{\omega_0^2}{s^2+\omega_0^2} \tag{5}$$

with $\zeta_p < \zeta_z < 1$. The notch filter by itself can be tuned to introduce enough high gain in the controller transfer function to produce a satisfactory tracking performance, but its cascade implementation with the resonant controller results in two desirable effects: null tracking error for the nominal frequency and gain reduction in high frequencies since the controller roll off frequency is -40 dB/dec. Henceforth the controller defined in (5) will be called Notch-Resonant Controller. Based on this formulation, the original resonant controller will be recovered for $\zeta_z = \zeta_p$.

Fig. 1 shows the Bode diagram of both (2) and (5) for $\omega_0 = 10$ rad/s and different values of ζ_z. As seen from the figure, the proposed controller adds robustness to variations of the frequency of interest by the "enlargement" of the high gain region of the bode plot as ζ_z increases. On the other hand, ζ_p has the contrary effect as depicted in Fig. 2, when the robustness is increased for smaller values of ζ_p. We can also point out that there is no noticeable gain in reducing this parameter below $\zeta_p = 0.01$.

It is also worth mentioning that a controller described by (6) can ensure perfect tracking and rejection for ramp-like signals since they can be expressed as a periodic signals with fundamental frequency $\omega_0 = 0$. Finally, relations (3) and (5) can be combined to define a Notch-Resonant Controller with multiple frequencies as follows

$$G_{mnr}(s) = \prod_{k=1}^{M} \left(\tfrac{s^2+2\zeta_z\omega_k s+\omega_k^2}{s^2+2\zeta_p\omega_k s+\omega_k^2} \cdot \tfrac{\omega_k^2}{s^2+\omega_k^2} \right) \tag{6}$$

which can be seen as a series interconnection of M Notch-Resonant Controllers tuned at each frequency of interest. The above controller is readily put in state space form for LMI based design in the section that follows.

B. State space formulation

One may represent (4) with resonance peak at ω_k in state space form by

$$\begin{array}{rcl} \dot{x}_{nk}(t) & = & A_{nk}x_{nk}(t) + B_n e_i(t) \\ y_{nk}(t) & = & C_{nk}x_{nk}(t) + D_n e_i(t) \end{array} \tag{7}$$

where $x_{nk}(t) \in \mathbb{R}^2$ is the notch state, $e_i(t) = r_i(t) - y_i(t),\ i = 1, \cdots, p$ is the $i-$th error channel and

$$A_{nk} = \begin{bmatrix} 0 & \omega_k \\ -\omega_k & -2\zeta_p\omega_k \end{bmatrix},\ B_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_{nk} = \begin{bmatrix} 0 & 2\omega_k(\zeta_z - \zeta_p) \end{bmatrix},\ D_n = 1.$$

In the same way, a resonant controller in (2) in series with the notch filter will be represented by

$$\begin{array}{rcl} \dot{x}_{rk}(t) & = & A_{rk}x_{rk}(t) + B_r y_{nk}(t) \\ y_{rk}(t) & = & C_r x_{rk}(t) \end{array} \tag{8}$$

Fig. 2. Resonant controller and the proposed Notch-Resonant Controller for different values of ζ_p and $\zeta_z = 0.1$.

with

$$A_{rk} = \begin{bmatrix} 0 & \omega_k \\ -\omega_k & 0 \end{bmatrix},\ B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_r = \begin{bmatrix} \omega_0 \\ 0 \end{bmatrix}'.$$

From both (7) and (8), it follows that (5) can be defined in terms of the augmented state $x_k(t) = \begin{bmatrix} x_{rk}(t)' & x_{nk}(t)' \end{bmatrix}' \in \mathbb{R}^4$ as

$$\begin{array}{rcl} \dot{x}_k(t) & = & A_k x_k(t) + \bar{B} e_i(t) \\ y_k(t) & = & x_k(t) \end{array} \tag{9}$$

where

$$A_k = \begin{bmatrix} A_{rk} & B_r C_{nk} \\ 0_{2\times 2} & A_{nk} \end{bmatrix},\ \bar{B} = \begin{bmatrix} B_r D_n \\ B_n \end{bmatrix}.$$

Consequently the parallel interconnection of M Notch-Resonant controllers (see Fig. 3) with frequencies $\omega_k,\ k = 1, 2, \cdots, M$ in (6) is represented in state space by

$$\dot{x}_{mn}(t) \quad = \quad A_{mn} x_{mn}(t) + B_{mn} e_i(t) \tag{10}$$

where $x_{mn}(t) \in \mathbb{R}^{4M}$ and

$$A_{mn} = \begin{bmatrix} A_1 & 0_{4\times 4} & \cdots & 0_{4\times 4} \\ 0_{4\times 4} & A_2 & \cdots & 0_{4\times 4} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{4\times 4} & 0_{4\times 4} & \cdots & A_M \end{bmatrix},\ B_{mnr} = \begin{bmatrix} \bar{B} \\ \bar{B} \\ \vdots \\ \bar{B} \end{bmatrix}$$

Finally, the MIMO nature of the open loop plant must be taken into account. Hence, to verify the IMP, the state space controller (10) must be inserted on each output error channel, resulting in

$$\dot{x}_c(t) \quad = \quad A_c x_c(t) + B_c e(t) \tag{11}$$

where $x_c(t) \in \mathbb{R}^{4Mp}$ and

$$A_c = diag\{\overbrace{A_{mn}, A_{mn}, \ldots, A_{mn}}^{p-tuple}\}$$

$$B_c = diag\{\overbrace{B_{mn}, B_{mn}, \ldots, B_{mn}}^{p-tuple}\}$$

In this case, (11) is a $4Mp-$dimensional equation where matrices A_{nm} and B_{nm} are repeated p times in a block diagonal structure [12]. To simplify notation along the paper we will consider $n_c = 4Mp$.

The augmented state space readily follows by defining $x_a(t) = [x(t)'\ x_c(t)']' \in \mathbb{R}^{n+n_c}$, giving rise to

$$\dot{x}_a(t) = (A_a + \Delta A_a(t))x_a(t) + B_a u(t) + B_q q(t) \tag{12}$$

where $q(t) = \begin{bmatrix} r(t)' & d(t)' \end{bmatrix}'$ and

$$A_a = \begin{bmatrix} A & 0_{n\times n_c} \\ -B_c C & A_c \end{bmatrix},\ \Delta A_a(t) = H_a \Xi(t) E_a$$

$$H_a = \begin{bmatrix} H_1 \\ -B_c H_2 \end{bmatrix},\ E_a = \begin{bmatrix} E & 0_{n\times n_c} \end{bmatrix}$$

$$B_a = \begin{bmatrix} B \\ 0_{n_c\times m} \end{bmatrix}\ B_q = \begin{bmatrix} 0_{n\times p} & B_d \\ B_c & 0_{n_c\times l} \end{bmatrix}$$

We may now consider the control law as a linear combination of plant, resonant and notch filter states such that

$$u(t) = K_p x(t) + K_c x_c(t) \tag{13}$$

with $K_c = [K_{c1}\ \ K_{c2}\ \ \cdots\ \ K_{cp}]$ and $K_{ci} = [K_{r1i}\ K_{n1i}\ K_{r2i}\ K_{n2i}\ \cdots\ K_{rMi}\ K_{nMi}],\ i = 1, 2, \cdots, p$. In this case, it follows that

$$u(t) = \begin{bmatrix} K_p & K_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} = K x_a(t). \tag{14}$$

Applying (14) to (12), the closed loop system is given by

$$\dot{x}_a(t) = (A_a + B_a K + \Delta A_a(t))x_a(t) + B_q q(t). \tag{15}$$

Remark 1: When the problem at hand involves only periodic and step-like signals, the introduction of a double integrator in the control loop as proposed in Section III-A may lead to stabilization problems. The usual solution in this case is to augment the controller state $x_{mn}(t)$ with a single integrator state given by

$$\dot{x}_i(t) \quad = \quad e_i(t), \tag{16}$$

i.e. $x_{mni}(t) = [x_{mn}(t)'\ x_i(t)']' \in \mathbb{R}^{4M+1}$. In the developments presented above, it suffices to replace $x_{mn}(t)$, A_{mn} and B_{mn} in (11) with $x_{mni}(t)$, $diag\{A_{mn}, 0\}$ and $[B'_{mn}\ 1]'$, respectively.

C. Stability Result

The first step towards our stability result is to deal with the vector of exogenous signals $q(t)$. Note that closed loop system (15) is linear and therefore its robust internal (asymptotic) stability also implies Bounded Input, Bounded Output (BIBO) stability. Thus, for stabilization purposes the term related to external signals $q(t)$ in (15) can be ignored, resulting in the following stabilization problem:

Problem 1: Determine a gain K such that

$$\dot{x}_a(t) = (A_a + B_a K + \Delta A_a(t))x_a(t) \tag{17}$$

is robustly asymptotically stable.

It is also important to point out that while the solution of Problem 1 deals only with the stabilization problem, the tracking/rejection problem is implicitly taken into account by the introduction of the resonant structure in the control loop. Hence, it leaves room for additional performance criteria such as:

PC1: Minimize the cost function

$$J(z(t)) := \|z(t)\|_2^2 = \int_0^\infty z(t)'z(t)dt \tag{18}$$

where $z(t)$ is a performance output defined by

$$z(t) := C_p x_a(t) + D_p u(t) \tag{19}$$

with $C_p,\ D_p$ being constant matrices with appropriate dimensions. By minimizing this cost function it is possible to penalize the control effort necessary

Fig. 3. Closed loop system for the $i-$th error channel.

to track/reject the periodic signal or the energy associated to augmented system states.

PC2: Ensure a given exponential decay rate α for the system trajectory such as:

$$\|x_a(t)\| \leq \beta \|x_a(0)\| \mathrm{e}^{-\alpha t}, \text{ for } t > 0 \tag{20}$$

where β is some positive scalar [13]. This restriction is directly related to the closed loop system transient response.

The following theorem is presented to solve Problem 1 and address the two criteria above:

Theorem 1: Suppose there exist a symmetric and positive definite matrix $Q \in \mathbb{R}^{(n+n_c)\times(n+n_c)}$, a matrix $Y \in \mathbb{R}^{m\times(n+n_c)}$ and the positive scalars ν and λ satisfying

$$\begin{bmatrix} \Lambda(Q,Y) & QE_a' & Y'D_p' + QC_p' \\ \star & -\nu I_{n_u} & 0_{nu\times m} \\ \star & \star & -\lambda I_m \end{bmatrix} < 0 \tag{21}$$

where $\Lambda(Q,Y) = A_aQ + Q'A_a' + B_aY + Y'B_a' + \nu H_aH_a' + 2\alpha Q$. Then, the closed-loop system in (17) with $K = YQ^{-1}$ is asymptotically stable with decay rate α and cost function (18) satisfies $\|z(t)\|_2^2 \leq \lambda x_a(0)'Q^{-1}x_a(0),\ t \geq 0$. □

The proof of Theorem 1 follows the same ideas presented in [13] and therefore will be omitted due to space constraints.

Note that for a given α, condition (21) is an LMI and, therefore, we can obtain controller gains that satisfy performance criteria PC1 and PC2 above by the solution of the following optimization problem:

$$\begin{gathered} \min_{Q,Y,\nu,\lambda} \ \lambda \\ \text{subject to (21).} \end{gathered} \tag{22}$$

IV. Numerical Examples

The numerical example considered is a MIMO plant borrowed from [14] (disregarding the saturation nonlinearity) whose dynamic matrices are given by

$$A = \begin{bmatrix} 0 & 10 & 0 & 1 \\ -100 & -30 & 0 & 0 \\ 0 & 0 & -37 & 1 \\ 0 & 0 & 0 & -19 \end{bmatrix},\ B = \begin{bmatrix} 0 & -51 \\ 17 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}$$

$$B_d = \begin{bmatrix} -1 & 7 \\ 11 & 0 \\ 3 & 23 \\ 1 & 0 \end{bmatrix},\ C = \begin{bmatrix} 10 & 20 & 0 & 0 \\ 0 & 10 & 0 & 0 \end{bmatrix},\ H_1 = \begin{bmatrix} 0 \\ 0 \\ 25 \\ 0 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 5 \\ 0 \end{bmatrix},\ E = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

For simulation purposes both references are considered step-like signals with amplitude $r_1 = 10$ and $r_2 = 5$. Disturbance $d_1(t)$ is a sinusoidal signal with frequency $\omega_{d_1} = 2\pi\sqrt{2}/2$, amplitude $d_1 = 10$ and assumes a value different from zero for $t \geq 11.2$s. In the same manner, disturbance $d_2(t)$ is a sinusoidal signal with frequency $\omega_{d_2} = 2\pi\sqrt{5}/5$, amplitude $d_2 = 5$ and starts to act when $t \geq 22.4$s. Finally, $\Xi(t)$ is assumed to be equal to zero for $t < 42$s and unitary otherwise.

Hence, based on the IMP, to ensure the perfect rejection of $d_1(t)$ and $d_2(t)$ it will be necessary to introduce a multiple resonant controller with frequencies ω_{d_1} and ω_{d_2} at each error channel. Also, an integrator is required to follow the step-like references. From the reasoning presented in Section II-B, the notch filter parameters necessary to implement the proposed approach were set to $\xi_z = 0.99$ and $\xi_p = 0.01$. With all parameters properly defined, controller gains were obtained from the solution of optimization problem (22) with $\alpha = 1$ and null matrices C_p and D_p.

In Fig. 4 the tracking error for each error channel when the disturbance frequencies are 5% greater than nominal values ω_{d_1} and ω_{d_2} is depicted. When no disturbance is acting ($0 \leq t < 11.2$s) the perfect reference tracking is achieved for both controllers. When disturbance $d_1(t)$ starts to act ($t = 11.2$s), the maximum tracking error in steady state for the Resonant Controller is around 2.3% while the one associated to the Notch-Resonant remains under 0.1%. When both disturbances are acting, the tracking error for the resonant jumps to 3.4% while for the notch-resonant it remains at 0.1%. Finally, a variation on the plant matrices results in a tracking error of 0.7% for the proposed controller which is 22 times smaller than the 16% obtained with the Resonant Controller. In Fig. 5, the simulated outputs and control signals are presented, where it is clear that the proposed controller achieves an improved tracking performance with almost the same control effort as the resonant. Our final result is presented in Table I where we compare the maximum steady-state tracking error associated with variations on the disturbance frequencies from 5% to 50%. For differences under 20% the proposed approach is capable to maintain the tracking error under 10% while the Resonant Controller exhibits error around 68%. For variations of 50% the tracking performance is heavily compromised for both controllers.

TABLE I

Maximum steady-state tracking error associated to variations on the disturbance frequency.

	Resonant		Notch-Resonant	
Freq. dev.	$e_{1max}[\%]$	$e_{2max}[\%]$	$e_{1max}[\%]$	$e_{2max}[\%]$
5%	6	16	0.2	0.7
10%	12	36	0.8	2.5
20%	24	68	3	9
50%	44	96	14	38

Fig. 5. Output and control signals for disturbances with frequency 5% greater than nominal values ω_{d_1} and ω_{d_2}.

Fig. 4. Tracking error for disturbances with frequency 5% greater than nominal values ω_{d_1} and ω_{d_2}.

V. Conclusion

In this work we developed a control structure that combined a Resonant Controller in series with a notch filter to improve the tracking performance under period variation. As presented in the Repetitive Control literature, the main idea was to enlarge the high gain region around the nominal resonance frequency. Notch filter parameters were determined from an analysis based on the controller frequency response, while the robust stability of the closed loop system was guaranteed by LMI conditions. The numerical example was a MIMO plant with step-like references and sinusoidal disturbances composed of non-multiple frequencies. It was possible to conclude that the proposed approach maintained the tracking error under 10% for frequency variations around 20% of its nominal value.

For future work we can point out the experimental validation of the proposed method in a rotating machine and a direct comparison with High Order Repetitive Controllers in terms of tracking performance, noise attenuation and minimum hardware requirements to implement the techniques.

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