AUTOMATING THE CHOICE OF THE REFERENCE MODEL FOR DATA-BASED CONTROL METHODS APPLIED TO PID CONTROLLERS

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Abstract— This work presents an algorithm that contributes to data-driven control design based on model reference control in order to make it more attractive from the industrial perspective. The algorithm consists in a procedure that automatically determines a reference model using control performance criteria (maximum overshoot, settling time) given by the user and some characteristics of the plant itself, identified using data collected from the process. This reference model can then be used by a data-driven control method to tune a PI/PID controller. Experimental results show the applicability of the algorithm.

Keywords— Model Reference control, PID controller, performance criteria, automation.

1 Introduction

Adaptive control has been a major field of research in control theory and successful applications have been developed over the past half-century (Bazanella et al., 2011). However, most quotidian industry applications do not seem to have assimilated this evolution. This gap between practical applications and the adaptive control theory, along with nonlinear behavior introduced by adaptation mechanisms, propelled a surge of interest in the data-driven alternative for controller's adaptation. Data-driven control differs from adaptive control essentially by the fact that parameter adjustments are always based on large batches of data rather than on a single input-output sample or a few samples. This means that the nonlinear behavior introduced by adaptation mechanisms is avoided, but there are other issues that still prevent these methods to be safely used by industrial applications.

One of the reasons is the fact that control performance criteria are usually based on continuous time domain response requirements, while the reference model used by data-driven control methods must be the translation of these requirements into a discrete time transfer function. To illustrate that, consider the case of settling time: it is known that the settling time is proportional to the dominant pole of the system. So, if one wants to determine a continuous transfer function for the desired response of the system, the dominant pole of the reference model needs to be chosen as the inverse of 1/4 or 1/5 of the settling time. However, when dealing with discrete time transfer functions, the pole position also depends on the sampling time, which means that a dominant pole in certain position can represent either a fast response or a slow one.

Another reason, and perhaps the most important one, is the fact that the whole theory of data-driven and adaptive control is based on the hypothesis that the controller class chosen to be tuned is such that the desired response is achievable. This means that the user needs to choose the controller class *and the reference model* to match this condition or at least get close to it. Matching the condition requires the knowledge of the process class (Bazanella et al., 2011; Eckhard et al., 2009). Without this knowledge, the designer needs to know some important characteristics of the process when choosing the reference model in order to obtain a response close to the desired one. If this is not considered then data-driven methods—especially the direct ones such as the Virtual Reference Feedback Method (VRFT) (Campi et al., 2002) and the Non-iterative Correlation based Tuning (CbT) (Karimi et al., 2007)—are still not safe to be used in industrial applications.

This work presents some contributions for reference model control methods, specially direct data-driven control methods, in order to make them more attractive for industrial applications. The automation of the reference model choice, based on an intermediate step of identification and ratios obtained through three performance criteria specified by the user, is the focus of this work. Besides, since PID controllers are widely found in industry (Ogata, 2009), our algorithm is simplified considering only these controllers and systems where they can achieve a good performance.

This paper is organized as follows. Section 2 presents some necessary definitions, while in Section 3 some control issues are presented such as the performance criteria addressed by our algorithm and some concerns on the reference model that will be derived from the procedure. Section 4 presents how the algorithm chooses the reference model between three different classes of models based on an identified model of the process and on the control criteria defined by the user. In

Section 5 the results of an application of a direct data-driven control method which uses the proposed algorithm to define the reference model are shown, and in Section 6 some conclusions and future work are presented.

2 Preliminaries

Consider a linear time-invariant discrete-time single-input-single-output process

$$\begin{aligned} y(t) &= G_0(z)u(t) + v(t) \\ &= G_0(z)u(t) + H_0(z)w(t), \end{aligned} \quad (1)$$

where z is the forward-shift operator, $G_0(z)$ is the process transfer function, $u(t)$ is the control input, $H_0(z)$ is the noise model, and $w(t)$ is zero mean white noise with variance σ_e^2. Both transfer functions, $G_0(z)$ and $H_0(z)$, are rational and causal.

The task is to tune the parameter vector $\rho \in \Re^d$ of a linear time-invariant controller $C(z,\rho)$ in order to achieve a desired closed-loop response. We assume that this controller belongs to a given user-specified controller class $\mathcal{C}$ such that $C(z,\rho)G_0(z)$ has positive relative degree for all $C(z,\rho) \in \mathcal{C}$; equivalently, the closed loop is not delay-free. The control action $u(t)$ can be written as

$$u(t) = C(z,\rho)(r(t) - y(t)), \quad (2)$$

where $r(t)$ is a reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise, that is $\bar{E}\left[r(t)w(s)\right] = 0 \; \forall t, s$, and

$$\bar{E}[f(t)] \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E[f(t)]$$

with $E[\cdot]$ denoting expectation (Ljung, 1999). The system (1)-(2) in closed loop becomes

$$\begin{aligned} y(t,\rho) &= T(z,\rho)r(t) + S(z,\rho)v(t) \\ T(z,\rho) &= \frac{C(z,\rho)G_0(z)}{1 + C(z,\rho)G_0(z)} \\ &= C(z,\rho)G_0(z)S(z,\rho) \\ S(z,\rho) &= \frac{1}{1 + C(z,\rho)G_0(z)} \end{aligned}$$

where the dependence on the controller parameter vector ρ is now made explicit in the output signal $y(t,\rho)$.

Model Reference control design consists of specifying a "desired" closed-loop transfer function $M(z)$, which is known as the *reference model*, and then solving the following optimization problem

$$\min_{\rho} J^{MR}(\rho) \quad (3)$$

$$J^{MR}(\rho) \triangleq \bar{E}\left[(T(z,\rho) - M(z))r(t)\right]^2. \quad (4)$$

The *optimal controller* is defined as $C(z,\rho^{MR})$ with

$$\rho^{MR} = \arg\min_{\rho} J^{MR}(\rho).$$

It is assumed that the user can collect a batch of data from the process (1)

$$Z^N = [u(1),\; y(1), \ldots, u(N),\; y(N)].$$

His/her task is then to estimate the *optimal parameters* of the controller $C(z,\rho^{MR})$ from these data.

3 Control Issues

3.1 Choice of controller structure

Data-driven control methods can be used to estimate a large variety of controllers. Most of them have the restriction of controllers being linear in the parameters, as is the case of (Hjalmarsson et al., 1998; Karimi et al., 2004; Campi et al., 2002), but there are also the possibility of using more general structures as pointed out in (Sala and Esparza, 2005a; van Heusden et al., 2011) and used in (Campestrini et al., 2012). For the proposed automated choice of the reference model, the focus is on PI and PID controllers, since they are linear in the parameters and for this reason can be tuned using different data-driven methods. Besides that, the integral action of PI/PID controllers leads to null steady-state error, which is usually a concern when designing a controller for reference tracking.

The control law of an ideal PID controller is given by

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt} \quad (5)$$

where the task of the designer is to tune K_p, K_i and K_d, the proportional, integral and derivative gains, respectively. The signal $u(t)$ is the control signal, which is applied to the process to be controlled, and $e(t)$ is the error between the reference signal and the output of the process.

Applying the Euler method to (5), the discrete PID controller $C(z) = U(z)/E(z)$ is given by

$$\begin{aligned} C(z,K_p,K_i,K_d) &= K_p + K_i T_s \frac{1}{1-z^{-1}} \\ &\quad + \frac{K_d}{T_s}(1-z^{-1}), \\ C(z,K_p,K_i,K_d) &= [K_p \; K_i \; K_d] \begin{bmatrix} 1 \\ \frac{T_s}{1-z^{-1}} \\ \frac{1-z^{-1}}{T_s} \end{bmatrix}, \\ C(z,\rho) &= \rho^T \bar{C}(z). \end{aligned} \quad (6)$$

where $\rho = [K_p \; K_i \; K_d]^T$ and T_s is the sampling time. Notice that, with the choice of $\bar{C}(z)$ made in (6), ρ is the vector containing exactly the continuous PID controller gains the user wants to find.

3.2 Performance criteria for control systems

In this work, the automated choice of the reference model will use three performance criteria largely found in the literature: zero steady-state error for reference tracking, maximum overshoot and settling time.

In order to achieve the reference tracking criteria, the controller used in the closed loop needs to contain the integral action if the process does not have it, which is the case of PI/PID controllers. Besides that, the reference model steady-state gain has to be equal to 1, i.e.,

$$M(z)|_{z=1} = 1. \tag{7}$$

The second criteria addressed here is the settling time. Considering the settling time to be the instant of time that the response achieves 98% of the steady-state value, for a first order continuous process it is calculated as

$$t_s = \frac{\ln(0.02)}{p_s} \approx \frac{-4}{p_s} \tag{8}$$

where p_s is the (continuous model) process pole, and for a second order under-damped continuous process it is calculated as

$$t_s = -\frac{\ln(0.02)\sqrt{1-\xi^2}}{\xi\omega_n} \approx \frac{4}{\xi\omega_n} \tag{9}$$

where ω_n is the natural frequency and ξ is the damping factor of the process (Ogata, 2009). Besides, the relation between discrete and continuous poles is given by

$$p_z = e^{p_s T_s}, \tag{10}$$

where p_z is the discrete pole. So, in order to obtain a response with the desired settling time, the reference model should have a dominant discrete pole

$$p_z = e^{\frac{-4}{t_s}T_s}$$

or a pair of complex poles where the relation

$$\xi\omega_n = e^{\frac{4}{t_s}T_s}$$

is respected.

Finally, there is the overshoot criteria. It is important to remember that the overshoot criteria is not a hard constraint, but rather an acceptable margin for the performance of the closed-loop system. If the design acceptable overshoot is greater than zero, then the reference model should have a dominant pair of complex poles.

Consider a second-order continuous-time system whose characteristic polynomial can be represented by $s^2 + 2\xi\omega_n s + {\omega_n}^2$ and whose roots are

$$p_{s_{1,2}} = -\xi\omega_n \pm \jmath\omega_n\sqrt{1-\xi^2}. \tag{11}$$

From systems theory (Ogata, 2009), it is known that the highest value of overshoot (herein it will be worked with values from 0 to 1 instead the percentage, without loss of generality) is given by

$$M_0 = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \tag{12}$$

whose damping factor is equivalently given by

$$\xi = \sqrt{\frac{\frac{\ln^2(M_0)}{\pi^2}}{1+\frac{\ln^2(M_0)}{\pi^2}}}. \tag{13}$$

That is, from a design value of maximum overshoot, one can determine the damping factor from (13) and, having the desired settling time, the natural frequency is obtained from (9). Based on these values, the continuous poles are given by (11) and using the relation (10), the discrete poles of a second order reference model are given by

$$\begin{aligned} p_{z_{1,2}} &= e^{-\xi\omega_n T_s \pm \jmath\omega_n T_s\sqrt{1-\xi^2}} \\ &= e^{-\xi\omega_n T_s}[cos(\omega_n T_s\sqrt{1-\xi^2}) \\ &\quad \pm \jmath sin(\omega_n T_s\sqrt{1-\xi^2})]. \end{aligned} \tag{14}$$

3.3 Reference model constraints

When a Model Reference control method is used, there are some constraints in the reference model that need to be considered. Since the controller structure is fixed and has zero relative degree (PI or PID), the reference model should have at least the same relative degree as the process. So, if the process presents a significant transport delay, it is important that the reference model also contains the process's transport delay. This is done trying to avoid high control signals which would probably saturate or even cause instability to the control loop.

Also, another undesirable situation is the cancellation of unstable poles and zeros. If the process contains non-minimum phase zeros, then the reference model must also contain them. Otherwise, the controller tends to cancel these zeros with unstable poles. Even if the controller's poles are fixed, the minimization of $J^{MR}(\rho)$ would probably cause instability to the control loop, as reported in (Campestrini et al., 2011).

Therefore, if the process model is unknown, the choice of the reference model requires an *a priori* knowledge of the transport delay of the process and the exact position of the non-minimum phase zeros, if they exist.

4 Automated choice of the reference model

From the section above, it can be seen that choosing a suitable reference model is not an easy task: requires some knowledge on the process and some experience from the designer. If the designer chooses a reference model that does not take into account the process characteristics, model reference control methods are not safe to be used in industrial applications. This may happen because, if the reference model specifies a performance that is too different from the best that can be achieved, then the closed loop with the obtained controller may have nothing to do with the desired response. Indeed, it can even be unstable.

In order to try to solve this problem, in this work an algorithm is proposed to automate the choice of the reference model based on some characteristics of the process that are going to be identified in a first step, using the same data collected for the controller design, and on some control criteria that are related to open-loop data.

In (Mårtensson and Hjalmarsson, 2009), the authors discuss how the location of poles and zeros of the system influence the quality of the parameters obtained by prediction error identification, as well as order and structure chosen for the model. They show that instability and inverse response (poles and zeros outside the unit circle) top the list of most easily identifiable characteristics and with less variance in the estimate, followed by large values of overshoot and by system's settling time.

Since there is no interest in finding a complete model for the system but only in some prevailing characteristics, the first step of the procedure is the identification of an AR(MA)X[1] model for the process with $G(z,\theta)$ given by

$$\begin{aligned} G(z,\theta) &= \frac{(\theta_1+\theta_2 z^{-1})z^{-nk}}{1+\theta_3 z^{-1}+\theta_4 z^{-2}} \\ &= \frac{K(1-fz^{-1})z^{-nk}}{(1-p_{z_1}z^{-1})(1-p_{z_2}z^{-1})}, \end{aligned} \quad (15)$$

which represents the open-loop behavior of the process. The idea is to vary nk from 1 to 10 and estimate AR(MA)X models using these different values. The model which gives the minimum value of $\bar{E}\left[y(t)-G(z,\theta)u(t)\right]^2$ is the one that will be used in the procedure. Notice that nk gives the estimate of the transport delay of the system. With this model one can also identify the non-minimum phase zero, if it exists. Also, as there are two poles to be identified, a complex pair of poles can be found. If this is not the case, i.e., the system does not present overshoot, then p_{z_1}, p_{z_2} will be real poles. In this case, it is assumed that $|p_{z_1}| \geq |p_{z_2}|$.

This model will be the source for the process characteristics that will be used in the reference model determination, as shown in the next subsections.

Notice that in theory only the issues treated in Section 3.3 need to be a concern when choosing the reference model, in order to prevent instability. However, since there is the open-loop settling time and overshoot (if exists) information (from the identified model), it is proposed to use them in order to prevent desired responses that are completely out of reach. To do so, the settling time criterion will be addressed as a percentage of speed considering the open-loop settling time. Also, the maximum overshoot criteria will be compared to the open-loop maximum overshoot, and the smaller between both will be used.

[1]Different structures could be chosen here. The focus is on ARX or ARMAX in order to further implement this algorithm in a micro-controller.

4.1 Determining reference model's dominant pole(s)

4.1.1 Real poles

Suppose one wants to make a closed-loop system $K_{\%}$ faster than in open loop, with no overshoot. Then the closed-loop dominant pole, or so, the reference model dominant pole with respect to the *identified* dominant pole in *continuous domain* (p_{s_1}) from the experiment should be such that

$$p_{s_{MR}} = \left(1+\frac{K_{\%}}{100}\right)p_{s_1}.$$

In discrete time this relation is established using (10) as follows

$$\begin{aligned} p_{MR} &= e^{p_{s_{MR}}T_s} \\ &= e^{p_{s_1}T_s\left(1+\frac{K_{\%}}{100}\right)} \\ &= p_{z_1}^{\left(1+\frac{K_{\%}}{100}\right)}. \end{aligned} \quad (16)$$

where p_{z_1} is the dominant pole of $G(z,\theta)$ (see (15)). If $G(z,\theta)$ presents a pair of complex poles instead of a dominant real pole, use the poles absolute value as $|p_{z_1}|$ to compute p_{MR}.

Mathematically, the range of $K_{\%}$ can be $[-99,\infty)$. Notice that negative values mean closed-loop responses which are slower than open-loop. If -100 is chosen, then poles equal to 1 will be set, and the system will present an integral behavior. On the other hand, in industrial applications a value of 100% faster than in open loop is already regarded as an adequate response.

Example: Suppose a process presents an open-loop response with settling time $t_s = 10$s. Considering that data are collected from this process with sampling time $T_s = 0.25$s. In continuous time, this process could be modeled with a dominant pole in $p_s = -0.3912$ which, with the adopted sampling time, represents a discrete pole in $p_z = 0.9068$. Hence, for a closed-loop system 20% faster than open loop, one can obtain the reference model dominant pole as

$$p_{MR} = 0.9068^{1.2} = 0.8892.$$

4.1.2 Pair of complex poles

Suppose now the designer allows the closed-loop response to present some overshoot. The idea is to diminish the open-loop overshoot or to maintain it, but not to increase its value. So, in order to obtain a reference model with these characteristics, the open-loop overshoot has to be estimated.

Suppose a process model $G(z,\theta)$ is obtained in the first step of the procedure, and its characteristic polynomial presents as roots a pair of complex poles (14). This polynomial can be written as

$$1+\theta_3 z^{-1}+\theta_4 z^{-2}=1 \\ -2e^{-\xi\omega_n T_s}cos(\omega_n T_s\sqrt{1-\xi^2})z^{-1}+e^{-2\xi\omega_n T_s}z^{-2}. \quad (17)$$

Let

$$l \triangleq \frac{\ln(\theta_4)}{2T_s}=-\xi\omega_n \quad (18)$$

$$m \triangleq \frac{\arccos\left(\frac{-\theta_3}{2\sqrt{\theta_4}}\right)}{Ts}=\omega_n\sqrt{1-\xi^2} \quad (19)$$

and

$$k \triangleq \frac{l}{m}=\frac{\ln(\theta_4)}{2\arccos\left(\frac{-\theta_3}{2\sqrt{\theta_4}}\right)}=-\frac{\xi}{\sqrt{1-\xi^2}}. \quad (20)$$

The similarity between (12) and (20) is evident, and so the maximum overshoot is determined from

$$M_0=e^{k\pi} \quad (21)$$

and the damping factor is given by

$$\xi=\sqrt{\frac{k^2}{1+k^2}}. \quad (22)$$

With these information, it is set as the *desired* overshoot M_{0_d} the smaller value between the one asked from the user and the open loop one, obtained from the identified model. Then, the *desired* damping factor ξ_d is computed using (22) if the system overshoot is smaller, or using (13) if the smaller is the one asked by the user. Once these quantities are chosen, there is left the settling time criteria to be determined. Consider making the closed loop being $K_\%$ faster. As shown in (9), the settling time depends only on the real part of the complex poles pair. Since the damping factor for the reference model has already been chosen, there is left only the *desired* natural frequency (ω_{n_d}) of the reference model to be set. So one have:

$$\omega_{n_d}=\frac{\left(1+\frac{K_\%}{100}\right)\xi\omega_n}{\xi_d}. \quad (23)$$

By using the relation (18) in (23) there is

$$\omega_{n_d}=\frac{-\left(1+\frac{K_\%}{100}\right)l}{\xi_d}. \quad (24)$$

Therefore, one have

$$l_2=-\xi_d\omega_{n_d} \text{ and } m_2=\omega_{n_d}\sqrt{1-\xi_d^2}$$

and the closed-loop poles are given by

$$p_{MR_{1,2}}=e^{l_2T_s}[cos(T_s m_2)\pm \jmath sin(T_s m_2)]. \quad (25)$$

Example: Let the denominator of an identified process, sampled at $T_s=0.05s$, be

$$1-1.4685159z^{-1}+0.6703200z^{-2}.$$

The performance criteria are $M_{0_d}=0.1$ and $K_\%=100$. First, the overshoot criteria must be verified. To do so, (21) is used to obtain $M_0=0.25$. Since the process overshoot is greater than the specification, the desired damping factor will be based on the desired $M_{0_d}=0.1$. The desired damping factor value is $\xi_d=0.59$, obtained from (13). Finally, using the performance criteria $K_\%$ and (24), one get $\omega_{n_p}=13.55$rad/s. Thus, the poles of the reference model are computed by (25), resulting in the denominator

$$1-1.1453295z^{-1}+0.4495761z^{-2}.$$

4.2 Determining the type and characteristics of the reference model

The automatic choice of the reference model will be based on three different reference models, whose parameters are going to be chosen according to process characteristics (obtained with the identified model) and control performance criteria defined by the user. All three models are parametrized to present a desired response with zero steady-state error. The reference models are described below:

- **Model 1:**

$$M_1(z)=\frac{1-a}{1-az^{-1}}z^{-nk}, \quad (26)$$

which represents a desired response with no overshoot. This model depends on the process's time delay, taken from the identified model, and the pole a, which is obtained from (16), depending on the dominant pole of the process and the settling time criteria.

- **Model 2:**

$$M_2(z)=\frac{(1-a)(1-b)}{(1-az^{-1})(1-bz^{-1})}z^{-nk} \quad (27)$$

which may represent a desired response with or without overshoot. This model depends on the process' time delay, taken from the identified model, and two poles, a and b. This poles are chosen as follows. If $M_{0_d}\neq 0$, then the process overshoot is identified from (21).

Using the smaller between these two values, it computes ξ_d from (13) and from the settling time criteria, computes ω_{n_d} using (24) to obtain a pair of complex poles (25). If $M_{0_d} = 0$, the algorithm uses (16) using the dominant pole of the process model (if it is a complex pair of poles both have same module) to define the dominant pole of the reference model. The second pole is chosen to be 4 times faster than the dominant pole. It is defined thereby a reference model with a dominant pole.

- **Model 3:**

$$M_3(z) = \frac{\frac{(1-a)(1-b)}{(1-c)}(1 - cz^{-1})}{(1 - az^{-1})(1 - bz^{-1})} z^{-nk} \quad (28)$$

which may represent a desired response with or without overshoot, and may contain a non-minimum phase zero. Herein, it will be considered non-minimum phase zeros only the positive ones, since the negative ones do not configure inverse responses, but instead inadequate sampling time (Åström et al., 1984). This model depends on the process's time delay, taken from the identified model, two poles, a and b, which are chosen as in **Model 2**, and a zero c, which is chosen as follows. If the zero of the identified process is non-minimum phase, then c is equal to this zero. If not, this zero is located in a position between the two poles of the model.

In order to use the algorithm, the designer needs to collect a batch of input and output data from the process, which cannot be purely steady-state data, but needs to contain at least a step change in the reference or a step disturbance. Besides, the user needs to define the control criteria speed $K_{\%}$ and maximum overshoot M_{0_d}. The choice between the three different models is then done automatically following the procedure described below.

1. Identify an AR(MA)X model of the process, where $G(z, \theta)$ is given as in (15);
2. Check if the identified zero f is non-minimum phase:
   (a) If yes, use **Model 3**;
   (b) If not, go to the next step;
3. Check if the identified poles are complex (have imaginary part):
   (a) If yes, use **Model 2**;
   (b) If not, go to the next step;
4. Determine whether dominance exists between real poles:
   (a) If not, use **Model 3**;
   (b) If yes, go to the next step;
5. Verify the design specification for maximum overshoot:
   (a) If $M_{0_d} \neq 0$, use **Model 2**;
   (b) If $M_{0_d} = 0$, use **Model 1**.

With the reference model chosen, the user is safe to apply a data-driven control method to tune the PI or PID controller.

5 Experimental Results

In order to evaluate the proposed algorithm, data were collected from a pilot plant in order to re-tune its controller based on given control performance criteria. The proposed algorithm calculates the reference model. A specific direct data-driven method called the Controller Identification method, presented in (Campestrini et al., 2012) was chosen to tune the controller. This method is based on writing the process model $G(z, \theta)$ in terms of the controller to be identified $C(z, \rho)$ and the known reference model $M(z)$. Thus, instead of identifying the process model, it directly identifies the controller that makes the closed loop as close as possible to $M(z)$.

An experiment was conducted in a spherical tank plant, which is presented in Figure 1. The plant is highly nonlinear and the level is controlled to be near the middle of the sphere, in order to minimize the nonlinear effect.

Figure 1: Pilot plant.

The controlled variable is the level in the upper left tank and the manipulated variable is the current at the inlet tank pump. The current signal is $4-20$mA standard and the tank level range is from 0 to 26cm (its diameter). As level plants have a nearly integrator characteristic (Campos and Teixeira, 2010), the experiment for data collection was done in closed loop. The sampling time is $T_s = 1$s and the initial controller was a PI whose discrete-time transfer function is given by:

$$C_{ini}(z) = \frac{4.5 - 3.8z^{-1}}{1 - z^{-1}} = [3.8\ 0.7] \begin{bmatrix} 1 \\ \frac{1}{1-z^{-1}} \end{bmatrix}. \quad (29)$$

The data collected for the procedure was obtained from the application of a reference signal which consisted in two steps with 4cm, one from 11cm to 15cm and the other in the opposite direction. The result of the performed test is shown in Figure 2.

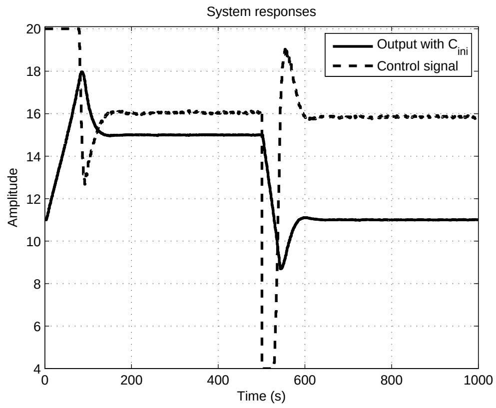

Figure 2: Closed-loop experiment in the level tank plant for the controller identification.

From this test, it is observed that the closed-loop system with the initial controller presents a response with a high overshoot. Much of this is due to the fact that the control signal has been left for a long period saturated at 20 mA. Based on this response, a PI controller was designed considering the following criteria: zero steady state error, $K_\% = 100$ faster than the open-loop response and $M_{0_d} = 0$. Notice that since a closed-loop experiment was conducted, there is no knowledge of the open-loop settling time. However, if the specification of $K_\% = 100$ results in a poor response (one can simulate the reference model response before applying the method), the specification can be redefined to obtain a better response.

From the collected data, the identified model for the construction of the reference model results in

$$G(z) = \frac{0.0016947(1 + 1.387z^{-1})z^{-1}}{(1 - 0.9888z^{-1})(1 - 0.587z^{-1})} \quad (30)$$

Using (30) and the control criteria defined by the user ($K_\% = 100$ $M_{0_d} = 0$), the algorithm chooses the reference model. Since the zero of the model is not a positive non-minimum phase zero, it is not included in the reference model. Then, the algorithm checks that the system is modeled with two real poles and there is clearly a dominant one. Also, since $M_{0_d} = 0$, the algorithm converges to **Model 1** and uses the design $K_\%$ to calculate the reference model as

$$M(z) = \frac{0.022197z^{-1}}{1 - 0.977803z^{-1}}. \quad (31)$$

Using this reference model and defining the controller class $C(z, \rho)$ as a PI controller, the Controller Identification Method results in

$$C_{id}(z, \rho) = \frac{2.3711(1 - 0.9896z^{-1})}{(1 - z^{-1})} = [2.3464\ 0.0247] \begin{bmatrix} 1 \\ \frac{1}{1-z^{-1}} \end{bmatrix} \quad (32)$$

which is then applied to the control system. The same reference signal applied for collecting data is applied now, with this new controller. The experiment result, compared with the desired response, is shown in Figure 3 and a comparison of the responses obtained with the controllers (29) and (32) is shown in Figure 4. The mean square error between the response of the system and the response of the reference model is $\epsilon = 0.0917$cm, while with the initial controller was $\epsilon = 0.1249$cm.

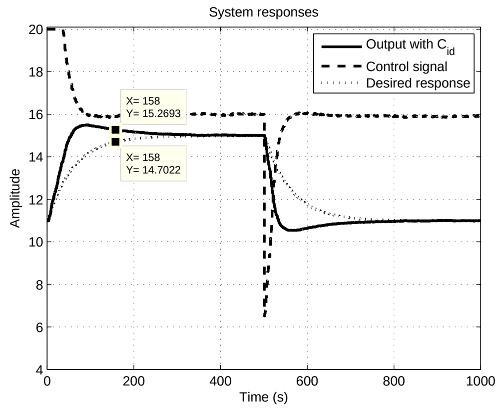

Figure 3: Comparison of the closed loop response obtained with the controller (32) and the desired response.

The closed loop system with the identified controller also presents overshoot, but significantly smaller compared to the one obtained with the initial controller, as shown in Figure 4. Once more, there was saturation of the control signal, but for a while almost 2 times smaller as can be seen comparing figures 2 and 3. Also, this was achieved just with better tuning of the controller and without any extra anti-windup procedure.

On the other hand, even with an undesired small overshoot and saturation in the control signal (as shown in Figure 3), the performance criteria $K_\%$ was attained. Indeed, in practical applications there is no guarantee that the desired

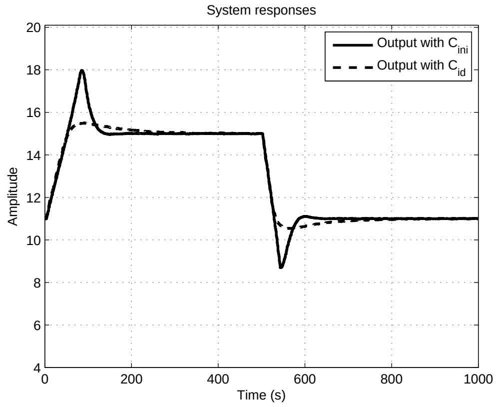

Figure 4: Comparison of the closed loop response obtained with the controllers (29) and (32).

response will be achieved. However, with the control performance choices being related to process characteristics, we can expect to obtain a close response to the desired one. The difference between the desired and the achieved response that can be seen in Figure 3 is mostly due to the non-linearity in the process gain (a variation of 4 cm in a 13 cm radius sphere), which decays until the level reaches the middle of the sphere and then starts to grow again.

6 Conclusions

A procedure that automatically calculates a discrete-time reference model from continuous-time performance criteria and from an intermediate step used to identify some characteristics of the process to be controlled were presented. Using this algorithm, the user needs only to determine if he/she wants to tune a PI or PID controller and define classical control performance criteria, like settling time and maximum overshoot. The user has no need to determine the reference model transfer function, which is determined by the procedure (order and location of poles and zeros). Also, in order to increase the possibility of achieving the desired response, the performance criteria are related to the open-loop response of the process.

The results obtained applying this procedure to a real plant are satisfactory, since the speed criterion is matched and the maximum overshoot has highly decreased.

The presented method was developed for tracking step references. As future work, it is considered to expand the algorithm to cope with other problems like tracking of sine waves, regulatory control and MIMO control problems.

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