Multivariable VRFT: an approach for systems with non-minimum phase transmission zeros

Gustavo R. Gonçalves da Silva and Lucíola Campestrini and Alexandre S. Bazanella

***Abstract*— In this paper we discuss an approach for the Virtual Reference Feedback Tuning (VRFT) for multivariable systems with non-minimum phase transmission zeros. We also present some possible choices for the reference model which satisfy the internal stability constraint. It is shown that with the addition of all-pass filters in the cost function one can find the optimal controller parameters without altering the cost function minimum and avoiding dealing with an unstable filter – which are two major limitations of the current multivariable VRFT approaches. Simulation examples show the applicability of the proposed formulation.**

***Index Terms*— VRFT, non-minimum phase transmission zeros, multivariable systems, PID control.**

I. Introduction

For Single-Input Single-Output (SISO) systems, the finite zeros are simply the roots of the system's transfer function numerator. If they are outside the unit circle (discrete-time case) they are called non-minimum phase (NMP) zeros. It is a well known result in control theory that when the loop is closed with a feedback controller the open-loop zeros remain unchanged. Thus, in a Model Reference context, if the system presents a NMP zero and the loop is closed with a feedback controller then the reference model must also have the NMP zero, otherwise internal stability can not be guaranteed, as the controller will tend to cancel out the NMP zero [1].

In a data-driven approach for the SISO Model Reference problem when the system presents a NMP zero, a solution based on the Virtual Reference Feedback Tuning (VRFT) method [2] was proposed in [3]. There, a flexible criterion for the reference model is used, and the resulting algorithm not only provides a tuned controller, but also identifies and includes in the reference model the NMP zeros, if any. When dealing with Multiple-Input Multiple-Output (MIMO) systems the definition of zeros, or in this case transmission zeros, and their implications on the closed-loop have to be generalized. In fact, for MIMO systems the transmission zeros also have input and output directions associated to them [4]. These directions have implications on output performance and allowable reference model choice.

In this paper it will be discussed how to extend the MIMO-VRFT method [5], [6], [7] to cope with NMP transmission zeros that have been incorporated into the reference model (and thus are known in advance). In this case the MIMO-VRFT method fails in computing the controller parameters, because the cost function depends on the inverse of reference model which then happens to be unstable; this limitation will be discussed later on.

We present an all-pass factorization for the multivariable discrete-time case, analogous to the continuous-time case presented in [4], which considers the system's transmission zeros input and output directions. Based on this factorization and on the structure of the reference model we show that, with the addition of some filters to the cost function, one can compute the controller parameters without dealing with an unstable filter and without altering the cost function minimum.

The paper is organized as follows: we start by stating the Model Reference control problem in Section II and also the constraint involving the NMP transmission zero. In Section III we review the VRFT approach and propose all-pass filters to solve the problem of using an unstable filter (the inverse of reference model). Simulation results are presented in Section IV and a conclusion is given in Section V.

II. Problem Statement

Consider a linear time-invariant discrete-time MIMO process

$$y(t) = G_0(q)u(t) + v(t), \tag{1}$$

where q is the forward-shift operator, $G_0(q)$ is the process transfer matrix, $u(t)$ is the control input vector and $v(t)$ is a noise vector. The transfer matrix $G_0(q)$ is a square $n \times n$ matrix whose elements are rational transfer functions. We also assume that every element of $G_0(q)$ is strictly proper.

The design task is to tune the parameter vector $P \in \mathbb{R}^p$ of a linear time-invariant controller $C(q, P)$ in order to achieve a desired closed-loop response. We assume that this controller belongs to a given user-specified controller class $\mathcal{C}$ such that all elements of $C(q, P)$ are proper. The control action $u(t)$ can be written as

$$u(t) = C(q, P)(r(t) - y(t)), \tag{2}$$

where $r(t)$ is the reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise. The system (1)-(2) in closed-loop becomes

$$\begin{aligned} y(t, P) &= T(q, P)r(t) + (I - T(q, P))v(t), & (3)\\ T(q, P) &= [I + G_0(q)C(q, P)]^{-1}G_0(q)C(q, P). & (4)\end{aligned}$$

The controller class $\mathcal{C}$ is defined as

$$\mathcal{C} = \{C(q, P) : P \in \Omega \subseteq \mathbb{R}^p\},$$

This work was supported by National Council for Technological and Scientific Development – CNPq/BR.{gustavo.rgs, bazanella}@ufrgs.br, luciola@ece.ufrgs.br.
The authors are with the Department of Automation and Energy, Federal University of Rio Grande do Sul, Porto Alegre, Brazil.

where the controller structure to be designed is defined as

$$C(q,P) = \begin{bmatrix} C_{11}(q,\rho_{11}) & C_{12}(q,\rho_{12}) & \cdots & C_{1n}(q,\rho_{1n}) \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1}(q,\rho_{n1}) & C_{n2}(q,\rho_{n2}) & \cdots & C_{nn}(q,\rho_{nn}) \end{bmatrix} \quad (5)$$

and $P = [\rho_{11}^T \;\; \rho_{12}^T \;\; \ldots \;\; \rho_{n1}^T \;\; \ldots \;\; \rho_{nn}^T]^T$.

In particular, the PID controller class with fixed derivative pole is such that each element in (5) has the structure

$$C_{ij}(q,\rho_{ij}) = \rho_{ij}^T \begin{bmatrix} 1 \\ \frac{q}{q-1} \\ \frac{q-1}{q} \end{bmatrix}. \quad (6)$$

In the Model Reference approach to the design, the closed-loop performance is specified through the desired closed-loop transfer matrix $T_d(q)$, also known as the *reference model*. The controller parameters are then tuned as the solution of the problem formally stated below.

$$\hat{P} = arg \min_P \quad J^{MR}(P) \quad (7)$$

$$J^{MR}(P) \triangleq \sum_{t=1}^{N} ||(T_d(q) - T(q,P))r(t)||_2^2, \quad (8)$$

where $r(t)$ is the reference signal of interest and N is the time horizon.

The *ideal controller* $C_d(q)$ is the one that allows the closed-loop system to match exactly $T_d(q)$ and is given by

$$C_d(q) = G_0(q)^{-1} L_d(q) \quad (9)$$

$$L_d(q) \triangleq T_d(q)[I - T_d(q)]^{-1}. \quad (10)$$

When the controller structure $\mathcal{C}$ is such that it can represent exactly the ideal controller $C_d(q)$ for some set of parameters, we say that *the ideal controller belongs to the controller class*.

Assumption 1: $C_d(q) \in \mathcal{C}$: There is a parameter vector P_0 such that $C(q,P_0) = C_d(q)$.

In a Model Reference design, if the system has a NMP transmission zero then an important constraint needs to be satisfied in order to obtain an ideal controller without poles outside the unit circle. In the remaining of this paper, we use the definitions of transmission zeros and their input $u_{z_{nm}}$ and output $y_{z_{nm}}$ direction as presented in [4].

Theorem 1 ([4]): If $G_0(q)$ is stable and has a non-minimum phase (NMP) transmission zero at z_{nm} with output direction $y_{z_{nm}}$, then for internal stability of the feedback system with the ideal controller, the following interpolation constraint must apply:

$$y_{z_{nm}}^H T_d(z_{nm}) = 0. \quad (11)$$

◇

In words, (11) says that $T_d(q)$ must have the same transmission zeros of $G_0(q)$ in the same output direction. It is important to notice that constraint (11) is a function of the *transmission zeros* and has no direct relation with the zeros of the elements of $G_0(q)$.

So if the system has non-minimum phase transmission zeros the user needs to know their location (just like in the SISO case [3]) in order to satisfy (11) and their direction may be necessary depending on the reference model structure, as will be seen in the next Section. It is out of the scope of this paper how the user can find these information, but it will be shown how to treat this particular case when using the VRFT to tune the controller and constraint (11) have already been incorporated in the reference model.

III. The VRFT approach

The VRFT method is a one-shot data based method, that is, with one batch of input-output data, the method searches for a controller that makes the closed-loop system as close as possible to the reference model. The main idea of the method is to find the minimum of $J^{MR}(P)$ criterion (8) without the knowledge of the process model and without using iterative algorithms. The user defines the reference model $T_d(q)$ and the controller structure, then the controller parameters are found through a least squares minimization.

A MIMO-VRFT derived directly from the SISO approach is presented in [7]. In their work the controller parameters are obtained by the solution of the following optimization problem:

$$\min_P \quad J^{VRF}(P)$$

$$J^{VRF}(P) = \sum_{t=1}^{N} \|F(q)[u(t) - C(q,P)\bar{e}(t)]\|_2^2, \quad (12)$$

where $u(t)$ and $\bar{e}(t) = (T_d(q)^{-1} - I)y(t)$ are vectors, $C(q,P)$ is the controller matrix and $F(q)$ is a filter that can be used as an additional degree of freedom by the user.

When the ideal controller $C_d(q)$ belongs to the chosen controller class, it is the minimum of $J^{VRF}(P)$, no matter which filter $F(q)$ is chosen. When this is not the case, the filter is chosen to approximate the minima of $J^{VRF}(P)$ and $J^{MR}(P)$. The filter is given by [7]

$$F(e^{\jmath\omega}) = T_d(e^{\jmath\omega})(I - T_d(e^{\jmath\omega}))\Phi_r^{1/2}(\omega)\Phi_u^{-1/2}(\omega), \quad \forall\omega \in [-\pi,\pi], \quad (13)$$

where $\Phi_r(\omega)$ and $\Phi_u(\omega)$ are the power spectra of the reference signal $r(t)$ we want to apply to the closed-loop system and the applied control signal $u(t)$ respectively, and $\Phi_x^{1/2}(\omega)$ denotes a spectral factor of $\Phi_x(\omega)$.

If $C(q,P)$ is linearly parametrized then $J^{VRF}(P)$ is quadratic in the parameters and a closed solution to the optimization problem is given by

$$\hat{P} = \left(\sum_{t=1}^{N} \varphi(t)\varphi^T(t) \right)^{-1} \sum_{t=1}^{N} \varphi(t)w(t), \quad (14)$$

where

$$w(t) = F(q)u(t), \quad \varphi(t) = [A_1 \; A_2 \; \cdots \; A_n],$$

$$A_x = \begin{bmatrix} F_{x1}E_x(t) \\ F_{x2}E_x(t) \\ \vdots \\ F_{xn}E_x(t) \end{bmatrix}, \quad E_x(t) = \begin{bmatrix} \bar{C}_{x1}(q)\bar{e}_1(t) \\ \bar{C}_{x2}(q)\bar{e}_2(t) \\ \vdots \\ \bar{C}_{xn}(q)\bar{e}_n(t) \end{bmatrix} \quad (15)$$

for $x = 1, 2, \cdots, n$. When data is affected by noise then instrumental variables (IV) must be used [1].

Inversion of $T_d(q)$ in (12) yields an unstable filter when $T_d(q)$ has a NMP transmission zero. The key-idea is then to add an all-pass filter which reflects this zero inside the unit circle, so the inversion yields a stable filter without altering the minimum of (12). We shall discuss the solutions based on some choices for the reference model. Our choices will consider that we are aiming to tune a PID controller (6).

Case 1: diagonal reference model

Let us consider first the case where a diagonal reference model has been chosen, that is

$$T_d(q) = \begin{bmatrix} t_{11}(q) & 0 & \dots & 0 \\ 0 & t_{22}(q) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & t_{nn}(q) \end{bmatrix}, \quad (16)$$

where each element has the form

$$t_{ii}(q) = \frac{\frac{(1-p_{1ii})(1-p_{2ii})}{(1-z_{nm})}(q - z_{nm})}{(q - p_{1ii}))(q - p_{2ii})} . \quad (17)$$

Since it can be seen as a choice of n SISO models, we can use equal approach to [8] for the choice of the elements. If no overshoot is allowed, choose $p_{1ii} = e^{-4/n_s}$ and choose $p_{2ii} = \frac{z_{nm}(1-p_{1ii})}{(z_{nm}-p_{1ii})}$. If some overshoot is allowed, then choose complex values for $p_{1,2}$, with $\text{Re}\{p\} = \frac{|p|^2+z_{nm}}{2z_{nm}}$ and $|p| = e^{-4/n_s}$. Here, n_s is the desired number of samples in the settling time. The choices proposed for the second pole of the reference model elements yield the poles of a PID (6) if computing the ideal controller (see (9)–(10)).

Notice that the NMP transmission zero is present in each element of (16). Thus, for a diagonal structure of the reference model one does not need to be concerned with the zero output direction, since constraint (11) will be satisfied because $T_d(z_{nm}) = 0$.

To apply the VRFT method, we need to add a filter $L_a(q)$ to (12) that both reflects the zero inside the unit circle and does not change the minimum of (12). In order to cope with the second necessity, we define

$$J_1(q, P)^{VRF}_{NMP} = \sum_{t=1}^{N} ||F(q)L_a(q)[u(t) - C(q, P)(T_d^{-1}(q) - I)y(t)]||_2^2. \quad (18)$$

Also, in order to commute the filter $L_a(q)$ with $C(q, P)$ so we can multiply it with $T_d^{-1}(q)$, then $L_a(q)$ must be a scalar function multiplying the identity matrix and it must cope with the first necessity.

Since every element $t_{ii}(q)$ of $T_d(q)$ has the plant's NMP transmission zeros, we must reflect every zero inside the unit circle, which can be properly achieved with a scalar function times the identity matrix. The scalar function is a well-known Blaschke function given by

$$f(q) = \frac{|z_{nm}|}{z_{nm}} \frac{z_{nm} - q}{1 - z_{nm}^* q},$$

where z_{nm}^* is the complex conjugate of z_{nm}, and a generalized filter that considers every NMP transmission zero in the system is given by

$$L_a(q) = I \prod_{i=1}^{N_z} \frac{|z_{nm_i}|}{z_{nm_i}} \frac{z_{nm_i} - q}{1 - z_{nm_i}^* q} , \quad (19)$$

where N_z is the number of different NMP transmission zeros.

Let

$$\overline{T}_d(q) \triangleq T_d(q)L_a(q)^{-1}. \quad (20)$$

Then

$$J_1(q, P)^{VRF}_{NMP} = \sum_{t=1}^{N} ||F(q)[L_a(q)u(t) - C(q, P)(\overline{T}_d^{-1}(q) - L_a(q))y(t)]||_2^2. \quad (21)$$

Notice that in (21), the VRFT filter $F(q)$ can still be used to approximate the minima of (8) and (21) when Assumption 1 is violated.

A. *Case 2: non-diagonal reference model*

Consider now the case where the reference model is not diagonal but it satisfies (11). In this case, the NMP transmission zero *of the reference model* will have an output direction equal to the process (11), but its input direction will be different of the process zero input direction. Albeit, since the reference model is a known transfer matrix (unlike the process model) the input-output direction can be computed via a SVD procedure [4].

One special case is when the reference model has a block-triangular structure. This structure allows a design where we can move the effect of the NMP transmission zero to a specific output. Let k be the this output. Thus, the reference model can be defined as

$$T_d(q) = \begin{bmatrix} T_{d_{11}}(q) & 0 & 0 & 0 & \dots & 0 \\ 0 & T_{d_{22}}(q) & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \dots & \vdots \\ T_{d_{k1}}(q) & T_{d_{k2}}(q) & \dots & T_{d_{kk}}(q) & \dots & T_{d_{kn}}(q) \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & T_{d_{nn}}(q) \end{bmatrix}. \quad (22)$$

The elements $T_{d_{jj}}(q), j \neq k$, can be chosen according to the desired performance using models of first or second-order. The element $T_{d_{kk}}(q)$ must have the NMP transmission zeros and its poles are chosen according to performance criteria. The other elements of row k should be chosen so that the ideal controller matches the PID class, or at least is closer to it. Using (22), the filter (10) is given by

$$L_d(q) = \begin{bmatrix} \frac{T_{d_{11}}(q)}{1-T_{d_{11}}(q)} & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{T_{d_{22}}(q)}{1-T_{d_{22}}(q)} & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \cdots & 0 \\ \frac{T_{d_{k1}}(q)}{\text{den}(L_{d_{jk}}(q))} & \frac{T_{d_{k2}}(q)}{\text{den}(L_{d_{jk}}(q))} & \cdots & \frac{T_{d_{kk}}(q)}{1-T_{d_{kk}}(q)} & \cdots & \frac{T_{d_{kn}}(q)}{\text{den}(L_{d_{jk}}(q))} \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \cdots & \frac{T_{d_{nn}}(q)}{1-T_{d_{nn}}(q)} \end{bmatrix}$$

where $\text{den}(L_{d_{jk}}(q)) = (1 - T_{d_{jj}}(q))(1 - T_{d_{kk}}(q))), \ j = 1, \ldots, n$ and $j \neq k$.

Since $C_d(q) = G_0^{-1}(q)L_d(q)$, the elements of $L_d(q)$ should at least contain the poles of the PID controller. To simplify, assume $T_{d_{kk}}(q)$ as a second-order model with the NMP zero and $T_{d_{jj}}(q)$ as a first-order model, both without delays. Then

$$L_{d_{jj}}(q) = \frac{1 - p_{1jj}}{q - 1}, \tag{23}$$

$$L_{d_{kj}}(q) = \frac{T_{d_{kj}}(q)}{\frac{(q-1)}{(q-p_{1jj})} \frac{(q-1)\left(q - \frac{z_{nm}(1-p_{1kk}-p_{2kk})+p_{1kk}p_{2kk}}{1-z_{nm}}\right)}{(q-p_{1kk})(q-p_{2kk})}}. \tag{24}$$

Notice that a proper choice on $T_{d_{kj}}(q)$ and $T_{d_{kk}}(q)$ will make (24) present the poles of a PID controller. More specifically, $T_{d_{kj}}(q)$ should have the poles of both $T_{d_{jj}}(q)$ and $T_{d_{kk}}(q)$, a zero at one and also the expression $\frac{z_{nm}(1-p_{1kk}-p_{2kk})+p_{1kk}p_{2kk}}{1-z_{nm}}$ should be zero, which is obtained as proposed in the diagonal reference model.

Consider the following choice for $T_{d_{kj}}(q)$:

$$\begin{aligned} T_{d_{kj}}(q) &= K_j \frac{(q-1)(q-z_{kj})}{(q-p_{1jj})(q-p_{1kk})(q-p_{2kk})} \\ &= K_j(q - z_{kj})\overline{T}_{d_{kj}}(q), \end{aligned} \tag{25}$$

where variables K_j and z_{kj} are (dependents) degrees of freedom that can be used to satisfy $y_{z_{nm}}^H T_d(z_{nm}) = 0$. This can be achieved as follows: choose a value for K_j (we recommend a value near the ratio between directions $|y_j/y_k|$); compute z_{kj} using

$$z_{kj} = z_{nm} + \frac{y_j T_{d_{jj}}(z_{nm})}{y_k K_j \overline{T}_{d_{kj}}(z_{nm})}. \tag{26}$$

There is a compromise between choices of K_j and z_{kj} and the maximum value expected in the output of $T_{d_{kj}}(q)$. Moreover, it is easier to move the effect of the NMP zero to the output where directionality is greater. If k is this output, then one can expect a lower interaction. Also, it is expected that with the recommended choice for K_j, expression (26) would result in a zero z_{kj} inside the unit circle, so the output is less deteriorated. Furthermore, since $\overline{T}_{d_{kj}}(q)$ has the poles of both $T_{d_{jj}}(q)$ and $T_{d_{kk}}(q)$, if every loop j is "faster" or have the same speed as loop k, then one can also expect lower interaction.

If we consider time-delay, then a similar approach can be applied, and again the user must be aware that the ideal controller will not be in the PID controller class.

As for the VRFT criterion we would like to add a filter to the left of (21), in order to keep the minimum unchanged and to maintain the order of operators and signals. Since the criterion involves the inverse of a minimum phase factor of $T_d(q)$ (i.e. $\overline{T}_d^{-1}(q)$), the input factorization $L_I(q)$ [9] of the reference model is a suitable choice. We will present the filter next.

Nevertheless, this filter is not diagonal so it can not be commuted in (21). Thus, we use the filter $L_a(q)$ along with the input-factor filter $L_I(q)$. The criterion to be minimized is defined by

$$\begin{aligned} J_2(q, P)_{NMP}^{VRF} = \sum_{t=1}^{N} ||F(q)L_I(q)[L_a(q)u(t) \\ - C(q, P)(\overline{T}_d^{-1}(q) - L_a(q))y(t)]||_2^2, \end{aligned} \tag{27}$$

where $L_a(q)$ is given by (19) and $L_I(q)$ is given by:

$$L_I(q) = \prod_{i=1}^{N_z}(I + \tfrac{(|z_{nm_i}|-1)(|z_{nm_i}|q+|z_{nm_i}|)}{z_{nm_i}-|z_{nm_i}|^2 q}\bar{u}_{z_{nm_i}}\bar{u}_{z_{nm_i}}^H), \tag{28}$$

where $\bar{u}_{z_{nm_i}}$ is calculated according to [9], albeit a discrete-time formulation has been used to fit the scope of this paper. We pinpoint that

$$T_d(q) = T_d^{mi}(q)L_I(q),$$

where $T_d^{mi}(q)$ is minimum phase with NMP transmission zeros of $T_d(q)$ reflected inside the unit circle and $L_I(q)$ is an all-pass filter.

IV. Simulation results

Consider the system given by

$$G_0(q) = \begin{bmatrix} \frac{q}{(q-0.9)(q-0.8)} & \frac{0.6}{(q-0.9)} \\ \frac{1}{(q-0.9)} & \frac{0.2}{(q-0.9)} \end{bmatrix} \tag{29}$$

which has a NMP transmission zero at $z_{nm} = 1.2$ with output direction $y_{z_{nm}} = [-\sqrt{10}/10 \ \ 3\sqrt{10}/10]^T$. We simulated the above model applying a PRBS of amplitude 1 (1260 samples) to its inputs and adding filtered white Gaussian noise $v(t) = H(q)e(t)$ to its outputs, with

$$H(q) = \begin{bmatrix} \frac{q^2}{(q-0.9)(q-0.8)} & \frac{q}{(q-0.9)} \\ \frac{q}{(q-0.9)} & \frac{q}{(q-0.9)} \end{bmatrix},$$

such that the SNR for each output with respect to $v(t)$ is around 20. Since the system is affected by noise, two experiments were conducted in order to use instrumental variables. Using the two sets of data we identified the following model

$$G_0(q, \hat{\theta}) = \begin{bmatrix} \frac{1.0582(q-0.04006)}{(q-0.9024)(q-0.7944)} & \frac{0.54377(q-0.8065)}{(q-0.9024)(q-0.7944)} \\ \frac{1.0127}{(q-0.8999)} & \frac{0.18349}{(q-0.8999)} \end{bmatrix}. \tag{30}$$

We used the identified model to estimate the NMP transmission zero: $\hat{z}_{nm} = 1.224$ and its output direction $\hat{y}_{z_{nm}} = [-0.325769 \ \ 0.945449]^T$. Given this information, two sets of designs will be discussed: first, we choose a diagonal structure for the reference model and then we compare the results when we use the estimated transmission zero and the actual transmission zero; next we use the same comparison but considering the situation where the effect of the NMP transmission zero is moved to output 2.

As performance criteria we would like the first loop to be at least 150% faster than the open-loop response and the second loop at least 100% faster. So we choose a dominant pole at $q = 0.75$ for loop 1 and one at $q = 0.8$ for loop 2. Also, the controller structure is a full PID matrix in every case.

In the first reference model the second pole of each element will be given by $p_2 = \frac{\hat{z}_{nm}(1-p_1)}{(\hat{z}_{nm}-p_1)}$., that is, we are aiming for a matched case between the ideal and the identified controller, although using an estimation for the transmission zero. The reference model is given by

$$T_{d_1}(q) = \begin{bmatrix} \frac{-0.39559(q-1.224)}{(q-0.75)(q-0.6456)} & 0 \\ 0 & \frac{-0.37738(q-1.224)}{(q-0.8)(q-0.5774)} \end{bmatrix}, \qquad (31)$$

and the filter $L_a(q)$ is given by $L_a(q) = \frac{q-1.224}{1-1.224q}$.

Using the same batch of data used to estimate the process model, we identify a full PID controller with the modified VRFT method (21) using IV, resulting in

$$C_1(q,\hat{P}) = \begin{bmatrix} \frac{0.22031(q-0.8941)(q-0.8424)}{q(q-1)} & \frac{-0.59322(q-0.9304)(q-0.7954)}{q(q-1)} \\ \frac{-1.0705(q-0.9055)(q-0.8014)}{q(q-1)} & \frac{1.0074(q-0.9062)(q+0.01245)}{q(q-1)} \end{bmatrix}. \qquad (32)$$

The reference model considering the system's actual transmission zero is given by

$$T_{d_2}(q) = \begin{bmatrix} \frac{-0.41666(q-1.2)}{(q-0.75)(q-0.6666)} & 0 \\ 0 & \frac{-0.4(q-1.2)}{(q-0.8)(q-0.6)} \end{bmatrix} \qquad (33)$$

to which the ideal controller is given by

$$C_{d_1}(q) = \begin{bmatrix} \frac{0.20833(q-0.9)(q-0.8)}{q(q-1)} & \frac{-0.6(q-0.9)(q-0.8)}{q(q-1)} \\ \frac{-1.0417(q-0.9)(q-0.8)}{q(q-1)} & \frac{(q-0.9)}{(q-1)} \end{bmatrix}. \qquad (34)$$

The filter $L_a(q)$ for this case is given by $L_a(q) = \frac{q-1.2}{1-1.2q}$ and using the same batch of data as before the identified controller with the VRFT method is

$$C_2(q,\hat{P}) = \begin{bmatrix} \frac{0.21431(q-0.8818)(q-0.8503)}{q(q-1)} & \frac{-0.61327(q-0.9161)(q-0.8141)}{q(q-1)} \\ \frac{-1.0354(q-0.8999)(q-0.8015)}{q(q-1)} & \frac{0.97545(q-0.9008)(q+0.01769)}{q(q-1)} \end{bmatrix}. \qquad (35)$$

The closed-loop response with controller (32) is shown in Fig. 1 and with controller (35) in Fig. 2. Simulations considered a filtered white Gaussian noise on the system outputs such that the SNR approaches the one in open-loop.

It is noticeable that both responses are very similar and present a high coupling in loop 1, even using the actual zero location in the reference model, which would allow the desired response (33) to be achieved. This happens because data is affected by noise and, although instrumental variables have been used, some variance error is still present.

Following the steps given in Subsection III-A and the information obtained with the identified process model, we designed a reference model intending to move the effect of the NMP transmission zero to output 2 while keeping time criteria for each output. The reference model is

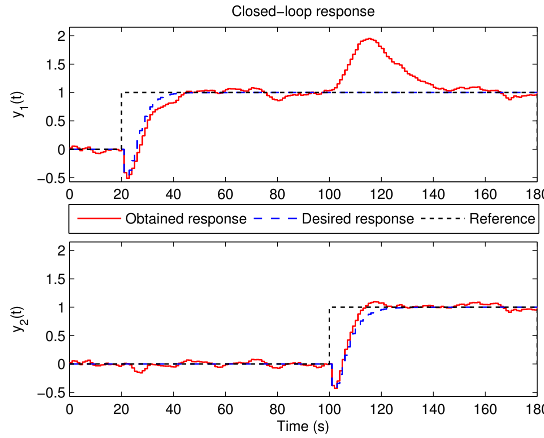

Fig. 1. Closed-loop response of (29) with controller (32).

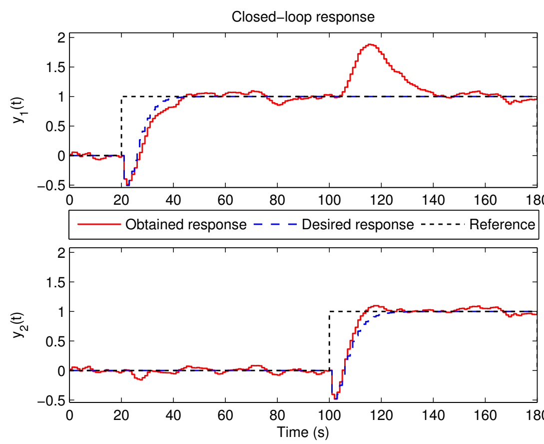

Fig. 2. Closed-loop response of (29) with controller (35).

$$T_{d_3}(q) = \begin{bmatrix} \frac{0.25}{(q-0.75)} & 0 \\ \frac{0.34457(q-1)(q-0.918)}{(q-0.8)(q-0.75)(q-0.5774)} & \frac{-0.37738(q-1.224)}{(q-0.8)(q-0.5774)} \end{bmatrix}. \qquad (36)$$

The input zero direction of *the reference model* is $\bar{u}_{z_{nm}} = [0 \quad 1]^T$, so the input filter is described by

$$L_I(q) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{-0.81701(q-1.1224)}{(q-0.81701)} \end{bmatrix}. \qquad (37)$$

Using the same batch of data as before, the identified controller is given by

$$C_3(q,\hat{P}) = \begin{bmatrix} \frac{0.51085(q^2-1.737q+0.7551)}{q(q-1)} & \frac{-0.86315(q^2-1.733q+0.7506)}{q(q-1)} \\ \frac{-0.31616(q-0.8997)(q-0.1868)}{q(q-1)} & \frac{1.1981(q-0.8993)(q-0.1603)}{q(q-1)} \end{bmatrix}. \qquad (38)$$

Now, performing the same design but using the actual values of the NMP transmission zero location and output direction, the reference model is

$$T_{d_4}(q) = \begin{bmatrix} \frac{0.25}{(q-0.75)} & 0 \\ \frac{0.3333(q-1)(q-0.9)}{(q-0.8)(q-0.75)(q-0.6)} & \frac{-0.4(q-1.2)}{(q-0.8)(q-0.6)} \end{bmatrix}, \qquad (39)$$

to which the ideal controller is given by

$$C_{d_2}(q) = \begin{bmatrix} \frac{0.375(q-0.9)(q-0.8)}{q(q-1)} & \frac{-0.6(q-0.9)(q-0.8)}{q(q-1)} \\ \frac{-0.20833(q-0.9)}{(q-1)} & \frac{(q-0.9)}{(q-1)} \end{bmatrix}. \quad (40)$$

The input filter for this case is

$$L_I(q) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{-0.8333(q-1.2)}{(q-0.8333)} \end{bmatrix}. \quad (41)$$

The identified controller was

$$C_4(q,\hat{P}) = \begin{bmatrix} \frac{0.4247(q^2-1.731q+0.7491)}{q(q-1)} & \frac{-0.72089(q^2-1.734q+0.7516)}{q(q-1)} \\ \frac{-0.21473(q-0.8974)(q-0.04837)}{q(q-1)} & \frac{1.0515(q-0.897)(q-0.06694)}{q(q-1)} \end{bmatrix}. \quad (42)$$

The closed-loop response with controller (38) is shown in Fig. 3 and with controller (42) in Fig. 4. Simulations considered a filtered white Gaussian noise on the system outputs such that the SNR approaches the one in open-loop.

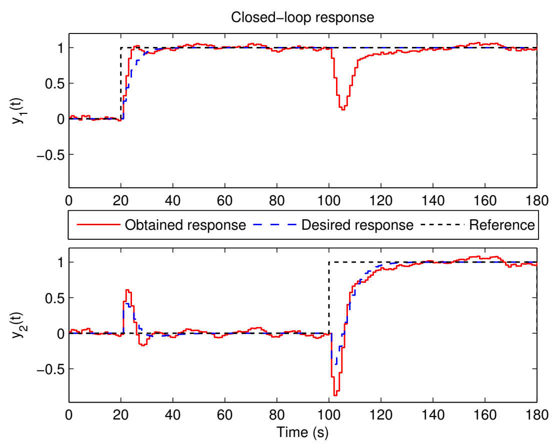

Fig. 3. Closed-loop response of (29) with controller (38).

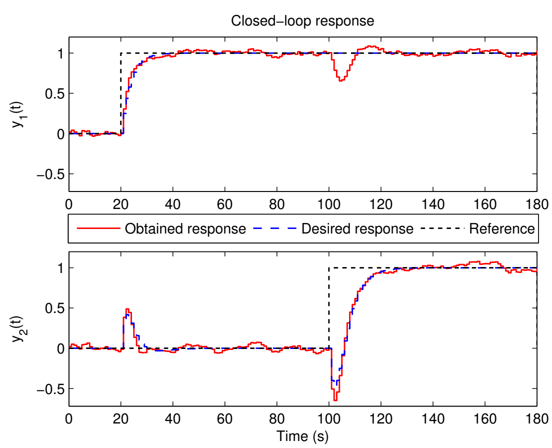

Fig. 4. Closed-loop response of (29) with controller (42).

For this case the difference between obtained responses is considerable. We see that estimation of the output direction plays an important role in the reference model choice which in turn affects the result obtained with the VRFT.

The estimated cost function (8) divided by the number of samples N considering controllers $C_1(q,\hat{P})$, $C_2(q,\hat{P})$, $C_3(q,\hat{P})$ and $C_4(q,\hat{P})$ are given respectively by: 0.2032, 0.1792, 0.1207 and 0.0682. It is noticeable that for a diagonal reference model choice the NMP transmission zero location can be estimated with a certain error margin and the obtained output with the identified controller is similar to the one when using the actual NMP transmission zero. We highlight, however, that the difference in the cost function values tends to be higher the less affected by noise the data is, causing the estimation using the VRFT to be better.

When going for a choice where the NMP transmission zero has a direction in the reference model though, estimation of the zero output direction is crucial for a better performance, whether data is affected by noise or not.

V. Conclusions

In this work we set out to provide a solution for the MIMO-VRFT when the system presents a NMP transmission zero. We showed that with the addition of two all-pass filters to the cost function one can avoid the problem of using an unstable filter (the inverse of reference model) and still find the optimal controller parameters.

Simulation results showed the applicability of the proposed formulation and stability was preserved, although the ideal controller could not always be found. We believe that because of the all-pass filters added to the cost function, a better formulation for the VRFT-filter $F(q)$ needs to be made for this particular case.

We point out that this work requires the knowledge of the NMP transmission zeros (and possibly their output direction). We aim next a MIMO approach using a flexible reference model similar to [3].

References

[1] A. S. Bazanella, L. Campestrini, and D. Eckhard, *Data-driven controller design: the H2 approach*. Netherlands: Springer Science & Business Media, 2012.

[2] M. Campi, A. Lecchini, and S. Savaresi, "Virtual reference feedback tuning: a direct method for the design of feedback controllers," *Automatica*, vol. 38, no. 8, pp. 1337 – 1346, 2002.

[3] L. Campestrini, D. Eckhard, M. Gevers, and A. S. Bazanella, "Virtual reference feedback tuning for non-minimum phase plants," *Automatica*, vol. 47, no. 8, pp. 1778–1784, 2011.

[4] K. Havre and S. Skogestad, "Effect of RHP zeros and poles on performance in multivariable systems," in *IEE Conference Publications*, vol. 2. London, England: IEEE, 1996, pp. 930–935.

[5] M. Nakamoto, "An application of the virtual reference feedback tuning for a MIMO process," in *SICE 2004 Annual Conference*, vol. 3. Sapporo, Japan: IEEE, 2004, pp. 2208–2213.

[6] S. Formentin and S. M. Savaresi, "Noniterative data-driven design of multivariable controllers," in *50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*. Orlando,FL: IEEE, 2011, pp. 5106–5111.

[7] L. Campestrini, D. Eckhard, L. A. Chía, and E. Boeira, "Unbiased MIMO VRFT with application to process control," *Journal of Process Control*, vol. 39, pp. 35 – 49, 2016.

[8] G. R. Gonçalves da Silva, L. Campestrini, and A. S. Bazanella, "Automating the choice of the reference model for data-based control methods applied to PID controllers," in *Proceedings of XX Congresso Brasileiro de Automática*. Belo Horizonte: SBA, 2014, pp. 1088–1095.

[9] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control Analysis and Design*, 2nd ed. Sussex: John Wiley & Sons, 2005.