Multivariable Virtual Reference Feedback Tuning for non-minimum phase plants

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***Abstract*—When applying model reference control to a non-minimum phase (NMP) plant, it is important to include the NMP (transmission) zero(s) in the reference model, otherwise the controller will tend to cancel out these zeros, often causing loss of internal stability. In data-driven (DD) control it is not possible to conceive *a priori* a reference model with the NMP zero(s) because no model of the plant is available; accordingly, DD design methods tend to fail in NMP plants. For SISO (single-input-single-output) plants, this problem has been solved in [1] by using a flexible reference model in a DD design. This paper presents an extension of that method for MIMO (multi-input-multi-output) plants.**

***Index Terms*—Sampled-data control, Identification for control, Numerical algorithms, Process Control.**

I. Introduction

DATA-DRIVEN (DD) control design methods provide solutions to classical control problems directly from data obtained from the system, contrasting with the more traditional model-based solutions. Most of the DD methods are based on the Model Reference approach to control design [2]. Virtual Reference Feedback Tuning [3] is one such method which by now has become a well-established tool for the design of reference tracking controllers.

A known limitation of the Model Reference approach, and thus of all the many design methodologies derived from it, lies in the control of non-minimum phase (NMP) plants. NMP behavior appears in a wide range of applications, from chemical processes to power systems and converters (see [4]), and is known to be responsible for critical performance limitations. Model Reference methods are likely to fail in these cases because the controller will tend to cancel out the plant's NMP zero(s), often leading to an unstable closed-loop, even when this nasty cancellation is not possible within the class of controllers considered in the design. Including the NMP zeros in the reference model is a solution that is well documented in the literature of direct adaptive control (see [5]), but requires knowing in advance the NMP zeros.

In [1] we have developed a DD method, based on VRFT, to deal with this issue in SISO plants. The method in [1] consists in setting a parametrized numerator for the reference model, which is to be estimated along with the controller parameters in an iterative optimization procedure. By doing so, the NMP zeros are identified without deriving a process model. A practical application of this method was presented in [6].

In this paper we develop a MIMO version of that method. The plant's transmission zeros are estimated along with optimal controller parameters, by means of a parametrized decoupled reference model; this is the first step of a two-step procedure. Once the NMP transmission zero(s) is(are) identified, a second step can be performed where the reference model is then fixed and VRFT is applied only to estimate new controller parameters that will provide enhanced performance.

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II. Preliminaries

Consider a linear time-invariant discrete-time MIMO process

$$y(t) = G_0(q)u(t) + v(t), \tag{1}$$

where q is the forward-shift operator, $u(t)$ and $y(t)$ are n-vectors representing the process' input and output, respectively, and the n-vector $v(t)$ is a stochastic process representing the noise. The transfer matrix $G_0(q)$ is a square $n \times n$ matrix whose elements are proper rational transfer functions.

The design task is to tune the parameter vector $P \in \mathbb{R}^p$ of a linear time-invariant controller $C(q, P)$ in order to achieve a desired closed-loop response. This controller belongs to a given controller class $\mathcal{C}$ such that all elements of the loop transfer matrix $L(q) = G_0(q)C(q, P)$ have positive relative degree for all $C(q, P) \in \mathcal{C}$. The control action $u(t)$ can be written as

$$u(t) = C(q, P)(r(t) - y(t)), \tag{2}$$

where $r(t)$ is the reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise $v(t)$ [7]. The system (1)–(2) in closed-loop becomes

$$\begin{aligned} y(t, P) &= T(q, P)r(t) + (I - T(q, P))v(t), && (3)\\ T(q, P) &= [I + G_0(q)C(q, P)]^{-1}G_0(q)C(q, P). && (4) \end{aligned}$$

In the Model Reference approach, closed-loop performance is specified through the desired closed-loop transfer matrix $T_d(q)$, also known as *reference model*. The controller parameters are then tuned as the solution of the problem formally stated below.

$$\hat{P} = arg \min_{P} \quad J^{MR}(P) \tag{5}$$

$$J^{MR}(P) \triangleq \sum_{t=1}^{N} ||(T_d(q) - T(q, P))r(t)||_2^2, \tag{6}$$

where $r(t)$ is the reference signal of interest and N is the time horizon.

The *ideal controller* $C_d(q)$ is the one that allows the closed-loop system behavior to match exactly the one prescribed by $T_d(q)$ and is given by

$$C_d(q) = G_0(q)^{-1} T_d(q)[I - T_d(q)]^{-1}. \quad (7)$$

If the ideal controller $C_d(q)$ were put in the control loop, the objective function $J^{MR}(P)$ would evaluate to zero, providing the ideal input-output performance. However, it is clear from (7) that the plant's zeros turn into poles of the ideal controller, which will result in internal instability for plants that possess NMP zeros. To avoid this destabilizing pole-zero cancellation, one should prescribe that the NMP zeros appearing in the denominator (7) are canceled by a proper choice of the reference model, as specified in the following Theorem.

Theorem 1 ([8]): If $G_0(q)$ has a non-minimum phase (NMP) transmission zero at z_{nm} with output direction $y_{z_{nm}}$, then for internal stability of the feedback system with the ideal controller, the following constraint must apply:

$$y^H_{z_{nm}} T_d(z_{nm}) = 0. \quad (8)$$

$\diamond$

It is clear – but still worth a remark – that constraint (8) is a function of the *transmission zeros* and has no direct relation with the zeros of the elements of $G_0(q)$. Theorem 1 states that in order to obtain internal stability, the reference model $T_d(q)$ must have the same NMP transmission zeros of $G_0(q)$ in the same output directions. So if the system has non-minimum phase transmission zeros the user needs to know at least their location (just like in the SISO case [1], [2]) in order to choose a reference model satisfying (8). As for the transmission zeros directions, there are two possibilities [9]. One is to choose the reference model such that the NMP transmission zero has the same direction as in the plant's transfer function; this of course requires knowledge of its direction in addition to knowledge of its value. Another choice is to put the NMP transmission zero in n linearly independent directions in the reference model, which is obtained putting it as a zero of each one of the elements in a diagonal $T_d(q)$, for instance. This second choice does not require knowledge of the NMP transmission zero's direction, but spreads its effect on the performance throughout all the outputs, and is the solution taken in this paper.

Data-driven control methods address the minimization of the criterion (6) directly from data collected from the system, without *a priori* knowledge of a process model and without deriving such a model from these data [10], [11], [12], [13], [14]. ***Since no process model is known, one can not assume prior knowledge of the NMP transmission zeros' locations, let alone their directions.*** As a consequence, direct application of such data-driven methods to NMP plants, whether SISO or MIMO, tends to fail because the reference model will lack the inclusion of the unknown NMP zeros. We have presented a solution to this problem in the SISO case, which represents an extension of the SISO-VRFT method [1]. In the present paper we extend this solution to the MIMO case by incorporating those ideas into the MIMO-VRFT.

The situation is similar to the SISO case, but presents additional challenges; let us briefly discuss some of them. In an SISO plant, the existence of an NMP zero is usually easy to detect from the plant's step response, since the NMP characteristic causes an inverse response in the first moments after the step.[1] This is not the case for NMP-MIMO plants, unless the input is applied in the NMP transmission zero's direction – a rare occurrence. Moreover, a step input in the MIMO case may produce an inverse response in a system that has **no** NMP transmission zeros. Another difficulty peculiar to the MIMO case is the one expressed in Theorem 1 and discussed in the paragraph following it: the reference model must consider not only the location of the NMP transmission zero in the complex plane, but also its direction. Conceiving an appropriate reference model $T_d(q)$ is also more involved in the MIMO case, an issue that we have studied in previous papers whose solutions we have applied to the examples in this paper [9]. Finally, a MIMO controller is bound to have a much larger number of parameters than an SISO controller, which raises numerical issues.

Let us first present the MIMO-VRFT problem and its usual solutions. The MIMO-VRFT approach was presented in [13]. A more general MIMO approach derived directly from SISO-VRFT has been presented in [14], in which the controller parameters are obtained by minimizing the following cost function:

$$J^{VR}(P) = \sum_{t=1}^{N} \|F(q)[u(t) - C(q,P)(T_d^{-1}(q) - I)y(t)]\|_2^2, \quad (9)$$

where $F(q)$ is a filter used to approximate the minima of (6) and (9) and the remaining variables have been previously defined. The filter used in this paper is:

$$F(q) = T_d(q)(I - T_d(q)). \quad (10)$$

If $C(q,P)$ is linearly parametrized, i.e., each element can be described as $c_{ij}(q) = \rho_{ij}^T \beta_{ij}(q)$ (which is the case of standard PID controllers), then $J^{VR}(P)$ is quadratic in the parameters and a closed-form solution to the optimization problem is obtained via least-squares [14]:

$$\hat{P} = \left(\sum_{t=1}^{N} \phi(t)\phi^T(t) \right)^{-1} \sum_{t=1}^{N} \phi(t) u_F(t), \quad (11)$$

where

$$u_F(t) = F(q)u(t), \quad \phi(t) = [A_1 \; A_2 \; \cdots \; A_n],$$

$$A_x = \begin{bmatrix} F_{x1}(q)E_x(t) \\ F_{x2}(q)E_x(t) \\ \vdots \\ F_{xn}(q)E_x(t) \end{bmatrix}, \quad E_x(t) = \begin{bmatrix} \beta_{x1}(q)\bar{e}_1(t) \\ \beta_{x2}(q)\bar{e}_2(t) \\ \vdots \\ \beta_{xn}(q)\bar{e}_n(t) \end{bmatrix} \quad (12)$$

for $x = 1, 2, \ldots, n$, where $\bar{e}(t) = (T_d^{-1}(q) - I)y(t)$ is the *virtual error*, $\bar{e}_i(t)$ is the $i-th$ component of $\bar{e}(t)$ and $F_{ij}(q)$ is the (i,j) element of the filter $F(q)$.

In the more practical situation in which the signals are noisy, the solution of the least-squares problem is biased and an

[1]This is true for plants with natural NMP behavior, but not necessarily when an NMP zero appears in $G(q)$ as a result of sampling phenomena.

instrumental variable technique should be used to eliminate the bias [2], [14].

As in any model reference design, MIMO-VRFT often results in an internally unstable closed-loop when the process is NMP and the reference model does not include the NMP transmission zeros. This instability tends to happen even when the controller class has fixed poles, for although the unstable pole-zero cancellation is not possible in this case, the design will still try (and to some extent succeed) to mimic the dreadful behavior of the ideal controller $C_d(q)$. Also, instability is more usual with NMP zeros that come from actual NMP behavior in continuous-time (which have positive real part) than with those which arise as a discretization phenomenon (which have negative real part).

An approach using VRFT, but which consider a previous step of process identification and inclusion of NMP transmission zeros in the reference model has been detailed in [9]. However, DD designs are most advantageous precisely because they do not require the identification of a model. An alternative solution is presented in this paper: the extension for the MIMO case of the method presented in [1], in which the use of a *flexible reference model* provides the NMP transmission zeros together with the controller parameters, without obtaining a full model for the plant.

III. The flexible performance criterion

The core idea of the proposed method is to use a two-step procedure, whose first step is solved using a parametrized transfer matrix reference model. The parametrization to be chosen will be the one most appropriated to the design method to be applied. In order to be able to apply a VRFT scheme similar to [1], the flexible reference model is chosen as

$$T_d(q) = T_d(q, \eta) = \eta^T \vartheta(q) I, \tag{13}$$

where $\eta \in \mathbb{R}^q$ is a vector of free parameters of the numerator of each element in $T_d(q, \eta)$ and $\vartheta(q)$ is a q-vector of proper rational functions.

In so doing, we are not specifying the whole transfer function; instead, degrees of freedom are left in its specification which can accommodate the inclusion of the necessary transmission zeros in the reference model. Notice that (13) implies that the same closed-loop behavior is specified for every input-output pair, which may be a restrictive performance choice. However, part of this drawback can be solved in the second step, which will be described later and explored in the examples. By choosing the reference model as in (13), we guarantee to satisfy (8) even without the information of the NMP transmission zeros directions. Moreover, this formulation keeps the attractive feature of VRFT that the estimates – in this case, both $\hat{\eta}$ and $\hat{P}$ – are simply obtained via least-squares.

The optimization of the flexible criterion is now made with respect to the controller parameters as well as the η parameters, which will provide the appropriate zeros in the reference model. We then have the following Assumption associated with the DD nature of the problem:

Assumption 1: There exists a pair (η^*, P^*) such that

$$C(q, P^*) = G_0^{-1}(q) T_d(q, \eta^*)[I - T_d(q, \eta^*)]^{-1}. \tag{14}$$

Under Assumption 1 and using (13), we have

$$(\eta^*, P^*) = \underset{\substack{\eta, P \\ (\eta, P) \neq \{0,0\}}}{\arg\min} \; \tilde{J}^{VR}(\eta, P) \tag{15}$$

$$\tilde{J}^{VR}(\eta, P) = \sum_{t=1}^{N} ||\bar{F}(q) T_d(q, \eta) u(t) - \bar{F}(q) C(q, P) \times (I - T_d(q, \eta)) y(t)||_2^2, \tag{16}$$

$$\bar{F}(q) = (I - T_d(q, \hat{\eta})). \tag{17}$$

The criterion $\tilde{J}^{VR}(\eta, P)$ in (16) is obtained by approximating the filter (10) as $F(q) = T_d(q, \eta)\bar{F}(q)$ and when the reference model is given as in (13), $T_d(q, \eta)$ commutes with all matrices and can be estimated in the procedure that will be described in the following. Given the linear parametrization of both the controller and the reference model, $\tilde{J}^{VR}(0, 0) = 0$, which is avoided by the constraint imposed in (15).

Since the argument in (16) is bilinear in η and P, the minimization of $\tilde{J}^{VR}(\eta, P)$ can be treated as a sequence of least squares problems [7]:

$$\hat{\eta}^{(i)} = \arg\min_{\eta} \; \tilde{J}^{VR}(\eta, \hat{P}^{(i-1)}) \tag{18}$$

$$\hat{P}^{(i)} = \arg\min_{P} \; \tilde{J}^{VR}(\hat{\eta}^{(i)}, P), \tag{19}$$

where we assume knowledge of one quantity (η or P) to obtain the other and the use of a known filter $\bar{F}(q)$. Each minimization has a least-squares solution. Assume that $C(q, \hat{P})$ is a known transfer matrix and insert (13) in (16). This gives

$$\begin{aligned}
\tilde{J}^{VR}(\eta, \hat{P}) &= \sum_{t=1}^{N} ||\eta^T \vartheta(q)[\bar{F}(q) u(t) + \bar{F}(q) C(q, \hat{P}) y(t)] \\
&\quad - \bar{F}(q) C(q, \hat{P}) y(t)||_2^2 \\
&= \sum_{t=1}^{N} ||\eta^T \vartheta(q) \bar{F}(q)[u(t) + C(q, \hat{P}) y(t)] \\
&\quad - \bar{F}(q) C(q, \hat{P}) y(t)||_2^2.
\end{aligned} \tag{20}$$

where $\bar{F}(q)$ can be obtained using the estimate of η obtained in the previous iteration, that is $\hat{\eta}^{(i-1)}$. Least-squares solution of (20) with respect to η is given by

$$\hat{\eta} = \left[\sum_{t=1}^{N} (\vartheta(q) w(t))^T (\vartheta(q) w(t))\right]^{-1} \left[\sum_{t=1}^{N} (\vartheta(q) w(t))^T \tilde{u}(t)\right] \tag{21}$$

where $w(t) = \bar{F}(q)[u(t) + C(q, \hat{P}) y(t)]$ and $\tilde{u}(t) = \bar{F}(q) C(q, \hat{P}) y(t)$.

Now, assume that $T(q, \hat{\eta})$ is a known transfer matrix. Then (16) can be rewritten as

$$\tilde{J}^{VR}(\hat{\eta}, P) = \sum_{t=1}^{N} ||T(q, \hat{\eta}) \bar{F}(q) u(t) - \bar{F}(q) C(q, P) \times (I - T(q, \hat{\eta})) y(t)||_2^2. \tag{22}$$

Least-squares solution to (22) is the same as (11) but with

$$u_F(t) = T(q, \hat{\eta}) \bar{F}(q) u(t) \quad \text{and} \quad \bar{e}(t) = (I - T(q, \hat{\eta})) y(t).$$

As in [1], since the procedure is iterative, initial values for $C(q, P^{(0)})$ and/or $T_d(q, \eta^{(0)})$ must be given. If data are collected in closed-loop then the first step of the sequential least squares is to identify the reference model. Remind that filter $\bar{F}(q)$ is a function of an estimate of η, which is unknown. In this case, we suggest to start the algorithm by using $\bar{F}(q) = I$ to obtain $\eta^{(1)}$ and then update the filter at each step using the obtained estimates of η. Finally it is important to highlight that, just as in the SISO case, even though the minimization algorithm is iterative, the data from the system are collected just once, thereby keeping the "one-shot" property of the VRFT method.

As mentioned before, minimization of $\tilde{J}^{VR}(\eta, P)$ corresponds to a diagonal reference model that contains the NMP transmission zeros of the plant, *if any*. We propose then a two-step procedure.

Step 1: Minimize $\tilde{J}^{VR}(\eta, P)$; call $(\hat{\eta}, \hat{P})$ the minimizing parameters and check the step response of $T_d(q, \hat{\eta})$. If it is satisfactory, apply $C(q, \hat{P})$ to the system. If not, go to *Step 2*.

Step 2: If the obtained $T_d(q, \hat{\eta})$ has NMP transmission zeros, then keep these zeros and change the reference model poles according to a desired response; if not, change poles and zeros accordingly. Apply the standard MIMO-VRFT.

Notice that while *Step 1* is used to identify the NMP transmission zeros, *Step 2* is used to eliminate the drawback of having the same desired response for every loop. However, we cannot eliminate the NMP effect from every output since we do not know its direction.

IV. Illustrative Examples

In this section we present simulation studies using the flexible VRFT scheme with the two-step procedure, for two different processes: one with an NMP transmission zero, and other with a minimum phase transmission zero. Besides, we also present an example where the collected signals are corrupted with noise.

A. Process with one NMP transmission zero

Consider a process described by

$$G_1(q) = \begin{bmatrix} \frac{(q-0.7)}{(q-0.9)(q-0.8)} & \frac{2}{(q-0.8)} \\ \frac{1.25}{(q-0.8)} & \frac{1.5}{(q-0.8)} \end{bmatrix}, \tag{23}$$

which has an NMP transmission zero at $q = 1.2$ with $y_{z_{nm}} = [-0.6 \quad 0.8]^T$. The plant's open-loop response to a sequence of steps shows no signs of initial inverse/oscillatory response and prior to any knowledge about the system's model, one might be tricked to choose a reference model without considering the NMP transmission zero, like the following desired reference model: $\widetilde{T}_d(q) = \frac{0.2}{(q-0.8)} I$. Applying the standard VRFT criterion to tune a PID controller yields

$$C(q,\hat{P}) = \begin{bmatrix} \frac{0.234(q+0.20)(q-0.91)}{q(q-1)} & \frac{-0.191(q+0.27)(q-0.86)}{q(q-1)} \\ \frac{-0.195(q+0.20)(q-0.91)}{q(q-1)} & \frac{0.293(q+0.15)(q-0.84)}{q(q-1)} \end{bmatrix},$$

which causes the closed-loop to be unstable, as the corresponding closed-loop transfer matrix will have poles at $q = 1.0105$. The design failed completely due to the absence of the NMP zero in the reference model. Before moving on, it is worth noting that this has occurred (as it typically does) despite the fact that the controller poles are fixed and thus cancellation of the NMP zero of the plant is not possible - see [1] for a discussion of this issue in the SISO case.

In order to cope properly with the NMP zero, we apply the proposed two-step procedure. Data is collected in closed-loop with an initial stabilizing proportional controller given by $C(q, \rho^{(0)}) = 0.5I$, with a sequence of steps as the reference signal. We study two situations: first the realistic situation in which Assumption 1 is violated, then the theoretical situation in which it is satisfied.

1) Assumption 1 is not satisfied: Consider the following flexible reference model

$$T_d(q, \eta) = \underbrace{[\eta_1 \quad \eta_2]}_{\eta^T} \underbrace{\begin{bmatrix} \frac{q}{(q-0.8)^2} & \frac{1}{(q-0.8)^2} \end{bmatrix}^T}_{\vartheta^T(q)} I, \tag{24}$$

for which Assumption 1 is not satisfied, which is the most common situation when the process model is unknown. Minimizing (16) using the iterative procedure (18)–(19) yields the following results at iteration 30:

$$T_d(q, \hat{\eta}^{(30)}) = \frac{-0.1884(q-1.212)}{(q-0.8)^2} I,$$

$$C(q,\hat{P}^{(30)}) = \begin{bmatrix} \frac{0.406(q-0.71)(q-0.92)}{q(q-1)} & \frac{-0.517(q-0.69)(q-0.92)}{q(q-1)} \\ \frac{-0.317(q-0.69)(q-0.92)}{q(q-1)} & \frac{0.280(q-0.64)(q-0.79)}{q(q-1)} \end{bmatrix}. \tag{25}$$

As indicated in [9], once the NMP transmission zero has been identified then a proper choice of the second pole based on the NMP transmission zero can be done. A suitable reference model choice in this case would be $T_d(q) = \frac{-0.388(q-1.212)}{(q-0.588)(q-0.8)} I$, which yields the controller

$$C(q,\hat{P}) = \begin{bmatrix} \frac{0.609(q-0.81)(q-0.9)}{q(q-1)} & \frac{-0.804(q-0.80)(q-0.90)}{q(q-1)} \\ \frac{-0.507(q-0.81)(q-0.9)}{q(q-1)} & \frac{0.411(q-0.72)(q-0.79)}{q(q-1)} \end{bmatrix}. \tag{26}$$

Fig. 1 shows a comparison between the response obtained with controllers (25) and (26) and their respective reference models. Some coupling between loops appears in the actual response for controller (25), specially from input 2 to output 1, whereas coupling has been much reduced with controller (26). For the first case the performance measure is evaluated as $J^{MR}(\hat{P}) = 5.95$ and for the second $J^{MR}(\hat{P}) = 1.70$, which is significantly smaller. Notice that we only changed the reference model, whereas the data were the same – no additional experiment was required for the redesign.

It is important to highlight here that even when the flexible reference model does not allow to achieve the matching condition (14), the NMP transmission zero can still be identified with good precision.

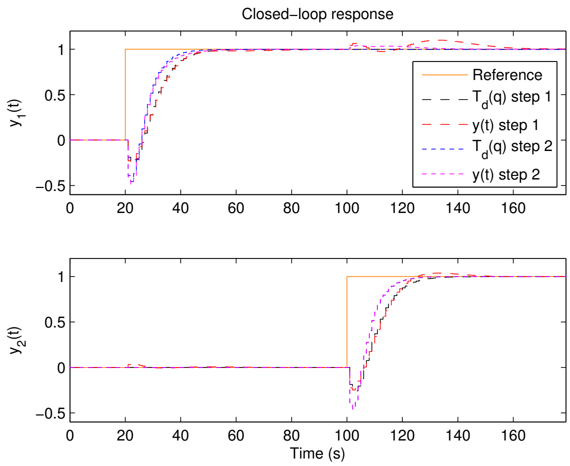

Fig. 1. Comparison of the closed-loop responses of system (23) with controllers (25) and (26).

2) Assumption 1 is satisfied: When Assumption 1 is satisfied, we know a priori what is the best controller: the one that provides exactly the performance that has been specified. Thus the study of this case allows to show that the method does converge to the optimal solution.

Consider the following flexible reference model $T_d(q,\eta) = \frac{\eta_1 q+\eta_2}{(q-0.8)(q-0.6)} I$, which satisfies Assumption 1 for a centralized PID controller.

We again minimize (16) using the iterative procedure (18)–(19). Table I shows the evolution of the estimated reference model parameters for some iterations, as well as the intermediate cost $\tilde{J}^{VR}(\eta, \hat{P}^{(i-1)})$ and the final cost $\tilde{J}^{VR}(\hat{\eta}^{(i)}, P)$.

TABLE I

EVOLUTION OF ESTIMATED NUM($T_d(q)$) AND VRFT COST FUNCTION

i	num($T_d(q)$)	$\tilde{J}^{VR}(\eta, \hat{P}^{(i-1)})$	$\tilde{J}^{VR}(\hat{\eta}^{(i)}, P)$
1	$0.13004(q-0.3848)$	67.764	6.497
2	$0.06105(q+0.3103)$	8.786	6.450
3	$-0.01139(q-8.0194)$	8.731	6.181
4	$-0.08420(q-1.9501)$	8.332	5.641
10	$-0.35951(q-1.2225)$	1.945	1.019
30	$-0.4(q-1.2000)$	2.1×10^{-4}	1.0×10^{-4}

At $i = 30$ we obtained $T_d(q, \hat{\eta}^{(30)}) = \frac{-0.4(q-1.2)}{(q-0.6)(q-0.8)} I$,

$$C(q, \hat{P}^{(30)}) = \begin{bmatrix} \frac{0.6(q-0.8)(q-0.9)}{q(q-1)} & \frac{-0.8(q-0.8)(q-0.9)}{q(q-1)} \\ \frac{-0.5(q-0.8)(q-0.9)}{q(q-1)} & \frac{0.4(q-0.7)(q-0.8)}{q(q-1)} \end{bmatrix}.$$

A good estimate of the NMP transmission zero is already obtained at iteration $i = 10$. Suppose now we would like to make loop 2 slower than loop 1, with a pole at $q = 0.9$. According to [9], we can properly choose the reference model as

$$\tilde{T}_d(q) = \begin{bmatrix} \frac{-0.4(q-1.2)}{(q-0.6)(q-0.8)} & 0 \\ 0 & \frac{-0.3(q-1.2)}{(q-0.4)(q-0.9)} \end{bmatrix}, \quad (27)$$

and proceed with Step 2 of the proposed methodology. The obtained controller is then

$$C(q, \hat{P}) = \begin{bmatrix} \frac{0.6(q-0.8)(q-0.9)}{q(q-1)} & \frac{-0.6(q-0.8)(q-0.9)}{q(q-1)} \\ \frac{-0.5(q-0.8)(q-0.9)}{q(q-1)} & \frac{0.3(q-0.7)(q-0.8)}{q(q-1)} \end{bmatrix}. \quad (28)$$

Fig. 2 shows the evolution of the reference model choices $\widetilde{T}_d(q)$, $T_d(q, \hat{\eta}^{(30)})$ and (27), and the closed-loop response of system (23) with controller (28), which has exactly the desired behavior (27) specified in Step 2.

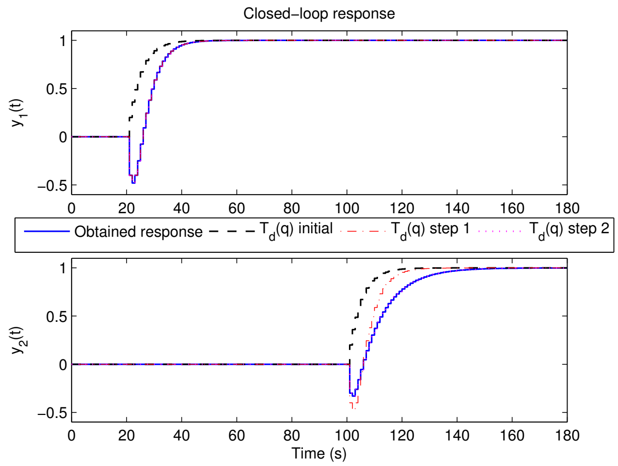

Fig. 2. Evolution of reference model choices and closed-loop response of system (23) with controller (28).

B. Process with one minimum phase transmission zero

Consider now a process described by

$$G_2(q) = \begin{bmatrix} \frac{25/24(q-0.7)}{(q-0.9)(q-0.8)} & \frac{0.625}{(q-0.8)} \\ \frac{0.4}{(q-0.8)} & \frac{1.2(q-0.78)}{(q-0.9)(q-0.8)} \end{bmatrix} \quad (29)$$

which has a minimum phase transmission zero at $q = 0.6$ with $y_z = [-0.755 \quad 0.655]^T$. A batch of data is obtained from a closed-loop experiment with $C(q, \rho^{(0)}) = 0.5I$ and we explore both cases considering the satisfaction of Assumption 1.

1) Assumption 1 is satisfied: Let the flexible reference model be $T_d(q,\eta) = \frac{\eta_1 q+\eta_2}{(q-0.85)(q+0.36)} I$, which satisfies Assumption 1. Minimization of (16) with 30 iterations resulted in:

$$T_d(q, \hat{\eta}^{(30)}) = \frac{0.5283(q-0.6138)}{(q-0.85)(q+0.36)} I,$$

$$C(q,\hat{P}^{(30)}) = \begin{bmatrix} \frac{0.632(q-0.90)(q-0.78)}{q(q-1)} & \frac{-0.33(q^2-1.805q+0.814)}{q(q-1)} \\ \frac{-0.21(q^2-1.807q+0.817)}{q(q-1)} & \frac{0.549(q-0.90)(q-0.69)}{q(q-1)} \end{bmatrix}.$$

Closing the loop with controller $C(q, \hat{P}^{(30)})$ and applying the same reference as presented in Fig. 1 yields a cost $J^{MR}(\hat{P}) = 1.6257 \times 10^{-4}$.

2) Assumption 1 is not satisfied: Let the flexible reference model be $T_d(q,\eta) = \frac{\eta_1 q+\eta_2}{(q-0.85)(q-0.4)} I$, which does not satisfy Assumption 1. Minimization of (16) with 30 iterations resulted in:

$$T_d(q, \hat{\eta}^{(30)}) = \frac{0.65615(q-0.8628)}{(q-0.85)(q-0.4)} I,$$

$$C(q,\hat{P}^{(30)}) = \begin{bmatrix} \frac{0.788(q-0.95)(q-0.13)}{q(q-1)} & \frac{-0.41(q-1.03)(q-0.20)}{q(q-1)} \\ \frac{-0.263(q-1.04)(q-0.20)}{q(q-1)} & \frac{0.684(q-0.90)(q-0.078)}{q(q-1)} \end{bmatrix}.$$

Notice that the identified transmission zero ($q = 0.8628$) is very different from the correct value ($q = 0.6$), whereas in the NMP example identification of the transmission zero was quite precise, even when Assumption 1 was violated. It is well known in SISO identification theory that NMP zeros are easier to identify than minimum-phase ones [15], and this example shows that this is also the case here. For lack of space we do not present the step responses for this example. However, closing the loop with controller $C(q, \hat{P}^{(30)})$ and applying the same reference as presented in Fig. 1 yields a cost $J^{MR}(\hat{P}) = 0.0483$. It is remarkable that, because minimum-phase zeros do not pose critical performance limitations, the resulting controller still provides closed-loop stability and a performance that is very close to the one specified.

C. The noisy unmatched-case

Consider again the NMP system (23) example, in closed-loop with the controller $C(q) = 0.5I$. We applied a PRBS with amplitude 1 and length of 1260 samples in the reference and the output is corrupted by white noise with $\sigma^2 = 0.025$ (SNR $\approx$ 25 dB) which represents $v(t)$ in (3). Consider also the parametrized reference model (24), representing the unmatched-case, which happens in the most practical cases. We performed 2000 Monte Carlo experiments (in order to estimate 1000 controllers using instrumental variables), with 70 iterations each[2] and the estimated zero is shown in Fig. 3.

From the experiments, 29 resulted in estimations of the NMP transmission zero with error larger than 5%; 225 resulted in estimations of the NMP transmission zero with error between 2–5%; and 746 resulted in estimations of the NMP transmission zero between a 2% margin around the actual value; all of them resulted in stable closed-loops. From this batch we get a mean value of $\mu_{\hat{z}_{nm}} = 1.215$, which is almost equal to the one obtained in the noiseless case. The standard deviation of the estimate was $\sigma_{\hat{z}_{nm}} = 0.0224$. Worst-case scenario, we obtained $\hat{z}_{nm} = 1.372$ with $J^{MR}(\hat{P}) = 24.1$.

V. Conclusions

Based on a previous procedure elaborated for SISO systems, we have extended the MIMO-VRFT method to cope with NMP multivariable plants. By means of a flexible reference model, the optimization is able to simultaneously identify NMP transmission zeros and include them in the reference model to find the optimal controller parameters, while keeping the "one-shot" characteristic of VRFT. The design procedure is purely data-driven – that is, model-free – and the case studies presented have shown that it provides convergence to the optimal controller, being applicable even when Assumption 1 is not satisfied (which represents most practical situations). The identification of the transmission zeros' directions with a similar data-driven procedure would allow to achieve better closed-loop performance and is a topic for future research.

[2]Our MatLab code takes less than 0.7 s for each simulation in a laptop PC, Intel Core i3, 4GB RAM; the whole Monte Carlo experimentation took 695 s.

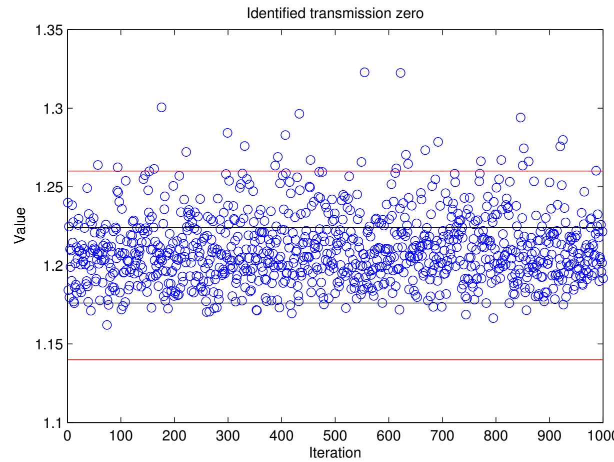

Fig. 3. Identified transmission zeros in 1000 Monte Carlo experiments; the 2% and 5% error margins are shown by the red and black horizontal lines, respectively.

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