

# Application of the Virtual Reference Feedback Tuning to a non-minimum phase pilot plant\*

Ricardo Scheid Filho,<sup>1</sup> Diego Eckhard,<sup>2</sup> Gustavo R. Gonçalves da Silva<sup>1</sup> and Lucíola Campestrini<sup>1</sup>

**Abstract**—Virtual Reference Feedback Tuning (VRFT) is a data-driven technique used to design controllers without the need of a process model, only input-output data is utilized. When the process has non-minimum phase zeros, the original method usually presents poor performance, because scarcely the reference model has the same non-minimum phase as the process. To overcome this problem, a flexible criterion has been proposed to the VRFT method, in a way that both the controller parameters and the NMP zero of the process are estimated together. In this paper we present the application of the VRFT method with flexible criterion to a level control MIMO pilot plant. We show that a sequential controller design may incorporate non-minimum phase behavior to the process. Then we use the VRFT method with flexible criterion to design the controller using only closed-loop data from the process.

## I. INTRODUCTION

In the past two decades, several data-driven control design methods have been proposed where a parameterized controller structure is chosen a priori, and the controller tuning is based directly on input and output data, without the need of a process model [1], [2], [3], [4], [5], [6]. Most of these methods seek for a controller to obtain a closed-loop response that is as close as possible to a *desired response*, considering reference signal changes. This desired closed loop response is the response of a *reference model*, which is chosen by the user.

When the controller structure is restricted, the successfulness of these methods relies on an adequate choice of the reference model. For that reason, even if the process model is unknown, some characteristics on the process need to be taken into account in order to obtain a “good” reference model: the non-minimum phase (NMP) zeros of the process being probably the most important characteristic to be known, besides an overbound on the relative degree of the process [7]. This means that, if the process is NMP, a “good” desired response is one that keeps the inverse response of the process, avoiding an unstable pole-zero cancellation with the controller to be designed. This adequate desired response is obtained with a reference model that contains the NMP zeros of the process.

\*This work was supported by CNPq - Conselho Nacional de Desenvolvimento Científico e Tecnológico.

<sup>1</sup>Ricardo Scheid Filho, Gustavo R. Gonçalves da Silva and Lucíola Campestrini are with Department of Automation and Energy, Federal University of Rio Grande do Sul, Porto Alegre, Brazil, [rscheid.filho@gmail.com](mailto:rscheid.filho@gmail.com), [gustavo.rgs@ufrgs.br](mailto:gustavo.rgs@ufrgs.br), [luciola@ece.ufrgs.br](mailto:luciola@ece.ufrgs.br)

<sup>2</sup>Diego Eckhard is with the Department of Pure and Applied Mathematics, Federal University of Rio Grande do Sul, Porto Alegre, Brazil, [diegoeck@ufrgs.br](mailto:diegoeck@ufrgs.br)

This becomes a problem when using data-driven methods to estimate the controller parameters, since the process model is unknown, and so is the NMP zeros. One possible solution to that is to first estimate the NMP zero of the process by an identification procedure; then include this zero in the reference model and use a data-driven method to obtain the controller parameters [8], [9]. Another possibility is to use a flexible reference model, where the controller parameters and the reference model numerator are identified together, in an iterative procedure: if the process is NMP, the numerator of the identified reference model will contain the NMP zero of the process by the end of the procedure and the estimated controller is the one to be used in closed loop. This methodology was first presented in [10] for the Iterative Feedback Tuning (IFT) method and later presented in [11] for the Virtual Reference Feedback (VRFT) method, where simulation results shows the efficiency of the method, but no experimental results have been provided.

In this paper, we present the results of the application of the methodology presented in [11] to a pilot plant. The plant is a two-input-two-output process, where the level of two tanks are controlled by two valves, in a decentralized control structure. We show that when a sequential tuning design is used, i.e. firstly one controller is designed and put in closed loop, and after the other controller is designed, the process may present non-minimum phase behavior. In this article we present how to choose the first controller in order to observe the inverse response on the second loop. Using this configuration, closed-loop experiments are run and the collect data is used to design new controllers applying the Virtual Reference Feedback Tuning with Flexible Criterion, without the need of a process model. The method is capable of both tune the controller and identify the NMP zero of the process. The obtained closed-loop response shows that the methodology can be applied to industrial processes.

The paper is organized as follows. Definitions and the problem formulation are presented in Section II. Section III reviews the standard and the flexible VRFT method, while the pilot plant is presented in Section IV, where bounds for a proportional controller in loop 1 are presented in order to obtain a NMP behavior in loop 2. Section V presents the experimental results of the flexible VRFT method application and some conclusions are presented in the end.

## II. PRELIMINARIES

Consider a linear time-invariant discrete-time single-input-single-output process

$$y(t) = G_0(q)u(t) + v(t), \quad (1)$$

where  $q$  is the forward-shift operator,  $G_0(q)$  is the process transfer function,  $u(t)$  is the control input and  $v(t)$  is a quasi-stationary noise process which can be written as  $v(t) = H_0(q)e(t)$  where  $e(t)$  is white noise with variance  $\sigma_e^2$ . Both transfer functions,  $G_0(q)$  and  $H_0(q)$ , are rational and causal and it is assumed that  $G_0(q)$  has a nonzero static gain.

This process is controlled by a linear time-invariant controller which belongs to a given user-specified controller class  $\mathcal{C}$  that is linearly parametrized:

$$\mathcal{C} : \{C(q, \rho) = \rho^T \beta(q), \rho \in \mathbb{R}^n\},$$

where  $\beta(q)$  is a  $n$ -column vector of fixed causal rational transfer functions, whose poles are strictly inside the unit circle except for possible poles at  $|q| = 1$ .

This class is such that  $C(q, \rho)G_0(q)$  has positive relative degree for all  $C(q, \rho) \in \mathcal{C}$ ; equivalently, the closed loop is not delay-free. The control action  $u(t)$  can be written as

$$u(t) = C(q, \rho)(r(t) - y(t)), \quad (2)$$

where  $r(t)$  is a reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise [12]. The system (1)-(2) in closed-loop becomes

$$\begin{aligned} y(t, \rho) &= T(q, \rho)r(t) + (1 - T(q, \rho))v(t) \\ T(q, \rho) &= \frac{C(q, \rho)G_0(q)}{1 + C(q, \rho)G_0(q)}. \end{aligned} \quad (3)$$

Model reference control design consists of specifying a “desired” closed-loop transfer function  $T_d(q)$ , which is known as the *reference model*, and then solving the following optimization problem

$$\min_{\rho} J^{MR}(\rho) \quad (4)$$

$$J^{MR}(\rho) \triangleq \|[T(q, \rho) - T_d(q)]r(t)\|_2^2. \quad (5)$$

When the plant is non-minimum phase (NMP), the model reference control design tends to produce unstable pole-zero cancellations, if the reference model  $T_d(q)$  does not have the same unstable zeros as the plant. So, the choice of the reference model requires *a priori* knowledge of the location of unstable zeros of the plant, if any [7], [8].

On the other hand, data-driven and direct adaptive control methods address the minimization of the criterion (5) directly from data collected from the system, without deriving a process model from this data [1], [2], [3], [4]. It is then not always possible to assume *a priori* knowledge of the existence of NMP zeros, and certainly not their positions in case they do exist. In this case, a data driven approach which identifies both the controller parameters and the process zero may be used. A method that uses this approach is the adapted version of the Virtual Reference Feedback Tuning, which is presented in the sequel.

### III. THE VRFT METHOD

#### A. The standard VRFT method

The Virtual Reference Feedback Tuning (VRFT) is a data-driven direct method proposed to provide an alternative for

controllers design when the process model is unknown [3]. The core idea of this method is to allow finding the controller parameters by minimizing a quadratic function, which only depends on the reference model and batches of data collected from the process, but no model of the plant is used.

Figure 1 shows the block diagram of the virtual closed-loop scheme of the VRFT. Through either an open-loop or closed-loop experiment, input data  $u(t)$  and output data  $y(t)$  from the process are collected. The virtual reference is defined such that

$$\bar{r}(t) = T_d^{-1}(q)y(t), \quad (6)$$

which gives the virtual error as

$$\bar{e}(t) = \bar{r}(t) - y(t). \quad (7)$$

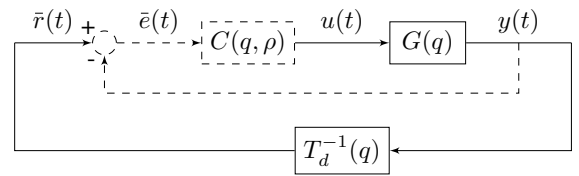


Fig. 1. Closed-loop block diagram and the virtual system's signals.

Notice that, with the knowledge of  $\bar{e}(t)$  and  $u(t)$ , the controller can be *identified* by solving the following optimization procedure

$$\min_{\rho} J^{VR}(\rho), \quad (8)$$

$$J^{VR}(\rho) = \|L(q)[u(t) - C(q, \rho)\bar{e}(t)]\|_2^2, \quad (9)$$

The filter  $L(q)$  is chosen to make the minimum of  $J^{VR}(\rho)$  as close as possible to the minimum of  $J^{MR}(\rho)$ , such that minimizing the convex function  $J^{VR}(\rho)$  produces similar result as minimizing the non-convex function  $J^{MR}(\rho)$  [3]. The filter is defined as

$$|L(e^{j\omega})|^2 = |T_d(e^{j\omega})|^2 |1 - T_d(e^{j\omega})|^2 \frac{\phi_r(e^{j\omega})}{\phi_u(e^{j\omega})}, \quad \forall \omega \in [-\pi, \pi]. \quad (10)$$

The cost function (9) is a quadratic function of  $\rho$  which can be solved through a least squares algorithm, as follows:

$$\hat{\rho} = \sum_{t=1}^N [\varphi_L(t)\varphi_L(t)^T]^{-1} \sum_{t=1}^N [\varphi_L(t)u_L(t)], \quad (11)$$

with  $N$  being the data length and

$$\varphi_L(t) = L(q)\beta(q)\bar{e}(t) \quad (12)$$

$$u_L(t) = L(q)u(t). \quad (13)$$

When the collected signals are noisy, an instrumental variable (IV) should be used in order to allow unbiased estimates [3]. The solution of (9) is then given by

$$\hat{\rho} = \sum_{t=1}^N [\zeta(t)\varphi_L(t)^T]^{-1} \sum_{t=1}^N [\zeta(t)u_L(t)], \quad (14)$$

where  $\varphi_L(t)$  is given by (12),

$$\zeta(t) = L(q)\beta(q) [T_d^{-1}(q) - 1] y'(t), \quad (15)$$

and  $y'(t)$  is the instrumental variable, which usually is the output data collected from a second experiment with the same input signal  $u(t)$  of the first experiment [3].

### B. The flexible criterion

When the system has a NMP zero that is not present in the reference model, the standard VRFT method present poor performance. In order to avoid this hurdle, a flexible criterion in which the identification of the NMP zeros is embedded into the VRFT method itself was proposed in [11].

The main idea of the flexible criterion, based on the solution proposed in [10] for the IFT method, is given by the use of a parametrized reference model such that the zeros are not fixed and can be defined as

$$T_d(q, \eta) = \eta^T F(q), \quad (16)$$

where  $\eta \in \mathbb{R}^d$  is a vector of free parameters and  $F(q)$  is a  $d$ -vector of transfer functions that establishes the reference model. Substitution of (16) in (9) yields, after some manipulations, a flexible virtual reference criterion, given by

$$J^{FVR}(\eta, \rho) = \left\| \eta^T F(q) [u_L(t) + \rho^T \beta(q) y_L(t)] - \rho^T \beta(q) y_L(t) \right\|_2^2, \quad (17)$$

where  $y_L(t) = L(q)y(t)$  and  $u_L(t) = L(q)u(t)$ .

The expression (17) is bi-quadratic in  $\eta$  and  $\rho$ , which means that for a fixed  $\rho_1$  the solution of the minimization for  $\eta$  can be achieved by least squares. Correspondingly, for a fixed  $\eta_1$  the solution for  $\rho$  can also be achieved by least squares [11]. Proceeding iteratively, at each iteration the following pair of least squares problem can be solved:

$$\eta_i = \arg \min_{\eta} J^{FVR}(\eta, \rho_{i-1}), \quad (18)$$

$$\rho_i = \arg \min_{\rho} J^{FVR}(\eta_i, \rho), \quad (19)$$

Since the procedure is iterative, initial values for  $C(q, \rho_0)$  and  $T(q, \eta_0)$  must be given. Also, the solution to the minimization problem can be treated as a sequence of least squares problems [7] where each minimization step has an explicit solution:

$$\hat{\eta}^i(\rho) = \sum_{t=1}^N \left\{ [F(q)w(\rho, t)] [F(q)w(\rho, t)]^T \right\}^{-1} \times \sum_{t=1}^N [F(q)w(\rho, t)] [C(q, \hat{\rho}^{i-1})y_L(t)],$$

where  $w(\rho, t) \triangleq [u_L(t) + C(q, \hat{\rho}^{i-1})y_L(t)]$  and the filter  $L(q)$  is a function of  $T(q, \hat{\eta}^{i-1})$ . The controller parameter

vector is estimated as

$$\hat{\rho}^i(\eta) = \sum_{t=1}^N \left\{ [\beta(q)\nu(\eta, t)] [\beta(q)\nu(\eta, t)]^T \right\}^{-1} \times \sum_{t=1}^N [\beta(q)\nu(\eta, t)] [T_d(q, \hat{\eta}^i)u_L(t)],$$

where  $\nu(\eta, t) \triangleq [1 - T_d(q, \hat{\eta}^i)] y_L(t)$  and the filter  $L(q)$  is a function of  $T(q, \hat{\eta}^i)$ .

Likewise the standard VRFT, the flexible criterion makes use of instrumental variables when data is affected by noise. In this case, two identical experiments are performed and the IV is formed using the input and output signals collected in the second experiment [7].

## IV. THE PILOT PLANT

The pilot plant is presented in Fig. 2 and its schematic diagram in Fig. 3 describes the process, which is built with off-the-shelf industrial equipments (pumps, valves, sensors and tanks). Tanks 1 and 2 have 70 liters each, while tank 3 is a 250 liters reservoir.



Fig. 2. Pilot plant.

Communication between devices is made up via a *Foundation Fieldbus H1* network [13]. The pumps are driven by frequency inverters and the valves are sliding stem pneumatic with embedded PID positioners. Level measurement is carried out by pressure sensors at the bottom of each tank and plant control and data acquisition is done in a supervisory

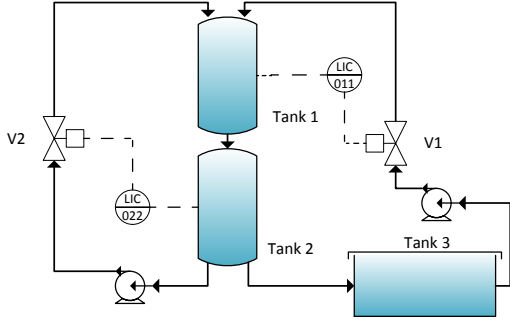


Fig. 3. Schematic diagram of the pilot plant.

software *Elipse SCADA* which communicates via an OPC server.

During all the experiments the frequencies of the inverters that drive the pumps are kept at a constant value while control is performed by valves  $V1$  and  $V2$ . System's multivariable behavior is represented by

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} G_{11}(q) & G_{12}(q) \\ G_{21}(q) & G_{22}(q) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}, \quad (20)$$

where  $y_1(t)$  and  $y_2(t)$  are the tanks' levels, and  $u_1(t)$  and  $u_2(t)$  are the valves' opening.

A decentralized structure for the multivariable controller is to be used, that is, two SISO PID controllers  $C_1(q)$  and  $C_2(q)$  should be tuned. The first controller  $C_1(q)$  controls the valve  $V1$  ( $u_1(t)$ ) by measuring the level in tank 1 ( $y_1(t)$ ), while controller  $C_2(q)$  controls the valve  $V2$  ( $u_2(t)$ ) by measuring the level in tank 2 ( $y_2(t)$ ).

We consider a sequential tuning to be used: firstly the controller  $C_1(q)$  is tuned using SISO tuning rules while loop 2 is kept in open loop. The controller  $C_1(q)$  is put in closed loop, and then  $C_2(q)$  is tuned, considering now the influence of loop 1 in loop 2. Observe that this is common practice in designing SISO controllers for MIMO processes. Also, this procedure may produce non-minimum phase behavior even to simple processes.

In this article, we will design controller  $C_1(q)$  for Pilot Plant aiming to obtain a non-minimum phase behavior. Notice that usually this is not desired, but it will be used only to explain how to use the flexible VRFT method and to validate the presented methodology. In order to obtain the non-minimum phase "desired" behavior, let us now assume that the level in tank 1 is controlled by a proportional feedback controller

$$u_1(t) = K_1(r_1(t) - y_1(t)),$$

where  $K_1$  is the proportional controller and  $r_1(t)$  is the reference signal for the level of tank 1, while the second loop is kept open. Then the level of the tank 1 can be written as

$$y_1(t) = G_{11}(q)K_1(r_1 - y_1(t)) + G_{12}(q)u_2(t),$$

or alternatively as

$$y_1(t) = \frac{G_{12}(q)}{1 + K_1G_{11}(q)}u_2(t) + \frac{K_1G_{11}(q)}{1 + K_1G_{11}(q)}r_1(t).$$

Now, the level of tank 2 can be written as

$$y_2(t) = G_{21}(q)K_1(r_1(t) - y_1(t)) + G_{22}(q)u_2(t)$$

and the transfer function from  $u_2(t)$  to  $y_2(t)$  is written as

$$G_{nmp}(q) = \left( G_{22}(q) - G_{21}(q)K_1 \frac{G_{12}(q)}{1 + K_1G_{11}(q)} \right).$$

Using first principles modeling, the system model can be represented by

$$G_0(q) = \begin{bmatrix} \frac{k_{11}}{q-p_1} & \frac{k_{12}}{q-p_1} \\ \frac{k_{21}}{(q-p_1)(q-p_2)} & \frac{-k_{22}(q-1)}{(q-p_1)(q-p_2)} \end{bmatrix}, \quad (21)$$

where  $k_{11}$ ,  $k_{12}$ ,  $k_{21}$ ,  $k_{22}$ ,  $p_1$  and  $p_2$  are positive constants. So, we can write

$$G_{nmp}(q) = \frac{-k_{22}q^2 + (k_{22} - \alpha)q + (\alpha - K_1k_{21}k_{12})}{(q-p_1)(q-p_2)(q-p_r)}, \quad (22)$$

where

$$\alpha \triangleq k_{22}(K_1k_{11} - p_1) \quad (23)$$

$$p_r \triangleq p_1 - K_1k_{11}. \quad (24)$$

In order to ensure that  $G_{nmp}(q)$  is stable, it is necessary that  $|p_r| < 1$  such that

$$\frac{p_1 - 1}{k_{11}} < K_1 < \frac{p_1 + 1}{k_{11}}. \quad (25)$$

And in order to ensure the process has at least one real non-minimum phase zero, we have that

$$\frac{-(k_{22} - \alpha) \pm \sqrt{(k_{22} - \alpha)^2 + 4k_{22}(\alpha - K_1k_{12}k_{21})}}{-2k_{22}} > 1,$$

which, after some algebraic manipulations, yields

$$K_1k_{12}k_{21}k_{22} < 0. \quad (26)$$

Since  $k_{11}$ ,  $k_{12}$ ,  $k_{21}$  and  $k_{22}$  are positive and the system is open loop stable, then the restrictions (25) and (26) can be written as

$$-\frac{1-p_1}{k_{11}} < K_1 < 0$$

or

$$-\frac{1}{G_{11}(1)} < K_1 < 0. \quad (27)$$

Therefore, if the user has an estimate of the DC gain of  $G_{11}(q)$ , then s/he can choose the gain  $K_1$  to ensure that  $G_{nmp}(q)$  is stable and non minimum phase.

In the next section we will present practical results used to choose controller  $K_1$ , in order to obtain a process  $G_{nmp}(q)$  which is stable and non-minimum phase. Then we will use the Flexible VRFT Criterion to design a controller to this NMP process, that does not uses a model for the process.

## V. EXPERIMENTAL RESULTS

### A. Inverse open-loop response

To obtain a non-minimum phase response in the second loop, a proportional controller  $K_1$  can be used to close loop 1. The plant was initially set to operate in open-loop and the valves  $V1$  and  $V2$  were open at 60% and 20% respectively. Once steady-state was reached, we could estimate  $G_{11}(1) \approx 1.1$ , which gives the bounds

$$-0.9 < K_1 < 0.$$

So, a robust choice for the proportional controller is  $K_1 = -0.5$ . The loop 1 was put in closed-loop with  $K_1 = -0.5$  while loop 2 was kept open. When steady-state was reached again, valve  $V2$  was open from 20% to 30%, performing a step change of 10%. Fig. 4 shows the level of Tank 2 when the step change in valve  $V2$  is applied, confirming the non-minimum phase behaviour. The open-loop settling time for this loop was estimated to 27 minutes.

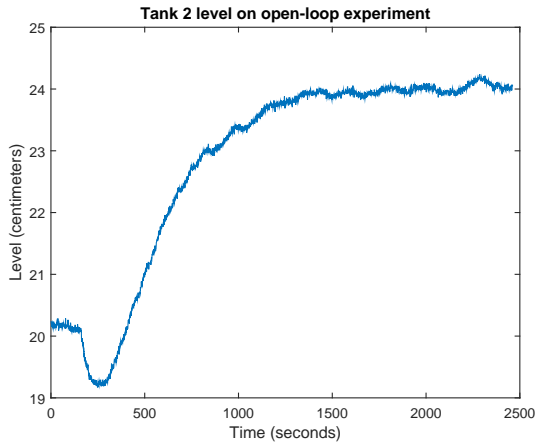


Fig. 4. Open-loop response for the loop 2, while controller loop 1 is closed with  $K_1 = -0.5$ .

### B. Application of the flexible VRFT

The desired performance for loop 2 is to obtain zero steady-state error and no overshoot for constant reference tracking. In order to do so, a PI controller is tuned:

$$C(q, \rho) = \begin{bmatrix} \rho_1 & \rho_2 \end{bmatrix} \begin{bmatrix} \frac{q}{q-1} \\ \frac{1}{q-1} \end{bmatrix}. \quad (28)$$

The flexible reference model was chosen as

$$T_d(q, \eta) = \begin{bmatrix} \eta_1 & \eta_2 \end{bmatrix} \begin{bmatrix} \frac{q}{(q-0.997)^2} \\ \frac{1}{(q-0.997)^2} \end{bmatrix}, \quad (29)$$

which would present a settling time approximately 32 minutes if it is disregarded the presence of zeros which can slower this settling time.

The flexible VRFT is based on an iterative algorithm to estimate the controller parameters and the reference model

numerator. For that reason, initial conditions are needed, as an initial controller and an initial reference model.

The initial controller was set to  $C_2(q) = 0.5$ , which yielded a stable closed-loop response. Also, an initial reference model is needed for the computation of the filter  $L(q)$  and is given by

$$T(q, \eta) = \frac{9 \times 10^{-6} q}{(q - 0.997)^2}. \quad (30)$$

Notice that the filter  $L(q)$  becomes a function of the flexible reference model computed in the latter step.

In order to obtain experimental data from the process, two closed-loop experiments were performed. In both experiments, the reference of loop 1 is kept constant while a step change is performed in loop 2. The controllers are initially proportional:  $K_1 = -0.5$  and  $C_2(q) = 0.5$  and the reference of loop 2 is a step signal from 14.3 cm to 24.3 cm. In Fig. 5 we can observe that the output signal is noisy and that the proportional controller can not achieve null steady-state error; in both experiments the level tank converged to approximately 16.7 cm. Both output signals are used to design the controller using the instrumental variable technique.

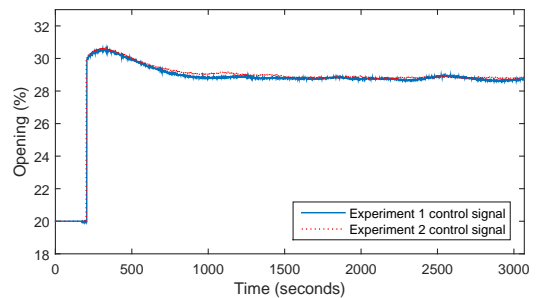
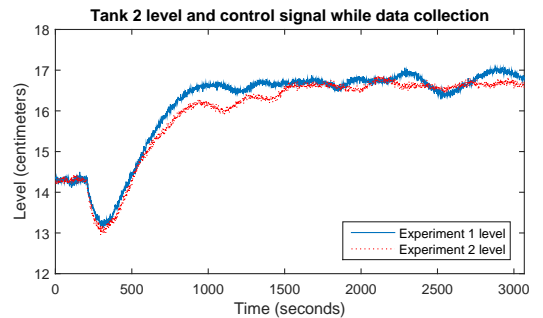


Fig. 5. Closed-loop responses with the initial proportional controller  $C_2(q) = 0.5$ .

With the defined reference model structure (29) and the controller structure (28), we applied the algorithm using the collected data. Table I shows the evolution of the controller parameters and flexible reference model parameters. Also, the  $J^{FVR}(\eta, \rho)$  is computed. We observe that the NMP zero begins to be identified already in the second iteration

TABLE I  
EVOLUTION OF THE COST FUNCTION MINIMIZATION AND PARAMETERS ESTIMATION.

iteration	$\text{num}(T(q, \hat{\eta}^{(i)}))$	$J^{FVR}(\hat{\eta}^{(i)}, \hat{\rho}^{(i-1)})$	$\text{num}(C(q, \hat{\rho}^{(i)}))$	$J^{FVR}(\hat{\eta}^{(i)}, \hat{\rho}^{(i)})$
1	$3.9104 \times 10^{-4}(q - 0.977)$	64.6815	$2.3414(q - 0.9979)$	73.2314
2	$-2.5274 \times 10^{-4}(q - 1.036)$	36.9911	$1.9528(q - 0.9976)$	38.8759
5	$-1.6919 \times 10^{-3}(q - 1.005)$	2.8880	$1.2376(q - 0.9967)$	3.5701
10	$-2.3093 \times 10^{-3}(q - 1.004)$	0.0327	$1.0181(q - 0.9961)$	0.1092
20	$-2.3346 \times 10^{-3}(q - 1.004)$	0.0664	$1.0101(q - 0.9961)$	0.0804
21	$-2.3346 \times 10^{-3}(q - 1.004)$	0.0664	$1.0101(q - 0.9961)$	0.0804
22	$-2.3347 \times 10^{-3}(q - 1.004)$	0.0665	$1.0101(q - 0.9961)$	0.0804

and from 20 to 22 iterations we notice convergence of the solution. The estimated reference model is then given by

$$T(q, \hat{\eta}_{22}) = \frac{-2.3347 \times 10^{-3}(q - 1.004)}{(q - 0.997)^2}, \quad (31)$$

whereas the tuned controller is given by

$$C(q, \hat{\rho}_{22}) = \frac{1.0101(q - 0.9961)}{(q - 1)}. \quad (32)$$

Once again, to evaluate the closed-loop response with the identified PI controller, the following experimental procedure was adopted: a closed-loop experiment was run with now the PI controller in the second loop. The reference of loop 1 is kept constant while the reference of loop 2 is a step signal from 9.33 cm to 19 cm. Fig. 6 shows the closed-loop response and the control signal with the estimated controller (32). Notice that the closed-loop response is close to the desired response (considering the presence of the NMP zero), specially in the inverse response, which shows that the NMP zero was successfully identified in the procedure. Now the closed-loop process present null steady-state error and the desired settling time.

## VI. CONCLUSIONS

In this work we set out to apply the flexible VRFT criterion for NMP plants proposed in [11] to a real pilot-plant. A typical NMP process behavior was obtained in the real plant by a sequential tuning of a decentralized controller in a MIMO plant: loop 1 was closed with a proportional controller and, with that loop closed, we performed an experiment to tune the controller in loop 2. Based on a parametric model of the plant, we have derived bounds on the proportional controller of loop 1 in order to obtain the “desired” inverse response in loop 2. Thus, we could validate the flexible VRFT in a real plant.

The controller was designed using two closed-loop experimental data, without the need of a process model. The closed-loop response with the controller obtained from the application of the flexible VRFT shows that we have successfully identified the NMP zero of the plant and that the designed PI controller is able to provide a closed loop response close to the response of the flexible reference model, with null steady-state error and desired settling time.

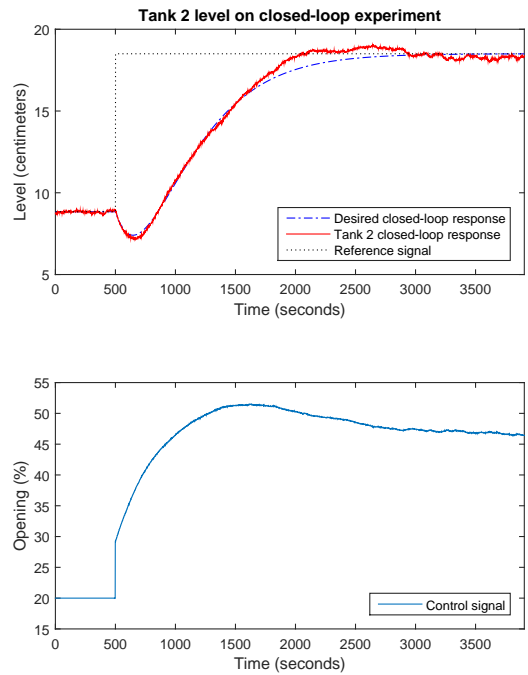


Fig. 6. Closed-loop response with the identified PI controller (32).

## REFERENCES

- [1] H. Hjalmarsson, M. Gevers, S. Gunnarsson, and O. Lequin, “Iterative feedback tuning: theory and applications,” *Control Systems, IEEE*, vol. 18, no. 4, pp. 26 – 41, 1998.
- [2] L. C. Kammer, R. R. Bitmead, and P. L. Bartlett, “Direct iterative tuning via spectral analysis,” *Automatica*, vol. 36, no. 9, pp. 1301–1307, 2000.
- [3] M. Campi, A. Lecchini, and S. Savaresi, “Virtual reference feedback tuning: a direct method for the design of feedback controllers,” *Automatica*, vol. 38, no. 8, pp. 1337 – 1346, 2002.
- [4] A. Karimi, L. Mišković, and D. Bonvin, “Iterative correlation-based controller tuning,” *International Journal of Adaptive Control and Signal Processing*, vol. 18, no. 8, pp. 645 – 664, 2004.
- [5] A. Karimi, K. van Heusden, and D. Bonvin, “Noniterative data-driven controller tuning using the correlation approach,” in *European Control Conference*. Kos Island, Greece: IEEE, 2007.
- [6] “Model reference control design by prediction error identification,” in *16th IFAC Symposium on System Identification*. Brussels, Belgium: IFAC, 2012, pp. 1478 – 1483.
- [7] *Data-driven controller design: the H2 approach*. Netherlands: Springer Science & Business Media, 2012.

- [8] A. Sala and A. Esparza, "Virtual Reference Feedback Tuning in restricted complexity controllers design of non-minimum phase systems," in *Proc. 16th IFAC World Congress*, Prague, Czech Republic, 2005, pp. 235–340.
- [9] "Automating the choice of the reference model for data-based control methods applied to PID controllers," in *Proceedings of XX Congresso Brasileiro de Automática*. Belo Horizonte: SBA, 2014, pp. 1088–1095.
- [10] A. Lecchini and M. Gevers, "On iterative feedback tuning for non-minimum phase plants," in *Decision and Control, 2002, Proceedings of the 41st IEEE Conference on*, vol. 4. IEEE, 2002, pp. 4658–4663.
- [11] L. Campestrini, D. Eckhard, M. Gevers, and A. S. Bazanella, "Virtual reference feedback tuning for non-minimum phase plants," *Automatica*, vol. 47, no. 8, pp. 1778–1784, 2011.
- [12] L. Ljung, *System identification: Theory for the user*, 2nd ed. New Jersey: Prentice Hall, 1999.
- [13] J. Gomes da Silva Jr., M. Silveira, and C. Pereira, "A web-based remote multivariable control experiment," in *11th IFAC Symposium on Information Control Problems in Manufacturing (INCOM'04)*, Salvador, 2004.