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Data-driven control design for load disturbance rejection by prediction error identification*

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***Abstract*— This paper presents a new direct data-driven control method for the load disturbance problem in a Model Reference Matching framework. It consists in embedding the controller's design under a prediction error approach, where a flexible reference model is also identified in order to guarantee the causality and stability of the ideal controller. Due to the complexity of the proposed approach, a dedicated iterative optimization algorithm is developed to properly solve the problem. Finally, the statistical properties of the obtained estimates are explored through simulation examples, where the enhancement obtained through the proposed methodology is compared to least-squares and instrumental variable solutions.**

I. INTRODUCTION

In most industrial applications of control systems, processes commonly possess coupled variables, in addition to possible unknown external influences, which can directly affect and disturb their nominal dynamics. Under this kind of environment, the disturbance rejection problem may be the most relevant issue since its occurrence is more frequent and harmful than a set-point change. Therefore, to obtain better load disturbance rejection performance, it is not appropriate to design the controllers employing a method formulated to reference tracking [1].

The literature on Model Reference Matching control for load disturbance rejection problem is more scarce and limited if compared to the reference tracking scenario. Concerning such problem, model-based approaches were presented at [2], [3], [4]. Moreover, in recent years, this problem has attained the attention of the data-driven control community, where efforts were made to conceive new methods aiming to enhance the performance of load disturbance attenuation. In [5], an approach based on the Virtual Reference Feedback Tuning (VRFT) method for continuous-time signals was presented to tune PID controllers. Also, in [6], [7], two and three degrees of freedom controllers are designed using an adaptation of the VRFT approach as well, but where it is necessary to measure the disturbance signal itself, which can be a challenging condition.

Eventually, an approach called Virtual Disturbance Feedback Tuning (VDFT), analogous to the VRFT solution, was developed in [1] to deal with data-driven load disturbance rejection. The great advantage of VDFT compared to its predecessors is that it allows the design of a wider set of controller's structure, and it does not need the measure of the disturbance signal specifically. Additionally, some novel contributions are arising in data-driven control literature to improve these methodologies. As an example, in [8], the choice of the load disturbance reference model was addressed for the specific scenario of data-driven control. Also, in [9] the VDFT was employed to tune regulatory controllers under a hierarchical model predictive control architecture, and in [10] the same method was extended to a correlation-based approach, focusing on the enhancement of its statistical properties.

Considering the load disturbance rejection problem in the data-driven control scenario, this work presents a new formulation of a design method for such context, extending the work of the VDFT, but embedding the problem on a prediction error approach. This solution is analogous to the one presented on [11] for the reference tracking problem (known as OCI), but with some evident new challenges, such as the solution of the optimization problem, which is evidently more complicated due to the structure and the dependence of the predictor on the controller's parameters. Because of this resemblance, the method proposed in this paper is named the *Optimal Controller Identification for Disturbance rejection*, or OCI-D.

II. SYSTEM PRELIMINARIES

Consider a linear time-invariant discrete-time single-input single-output process, described as [12]:

$$y(t) = G_0(q)u(t) + H_0(q)w(t), \tag{1}$$

where q is the forward-shift operator, $G_0(q)$ represents the process' transfer function, $H_0(q)$ is the noise transfer function, $w(t)$ is zero mean white noise, and $u(t)$ is the system's input signal, composed by two terms [1]:

$$u(t) = u_c(t) + d(t), \tag{2}$$

where $d(t)$ is the load disturbance signal and $u_c(t)$ is the control signal, that can be manipulated by the user through a feedback controller:

$$u_c(t,\rho) = C(q,\rho)\left(r(t) - y(t)\right), \tag{3}$$

where $r(t)$ is an external reference signal, and $C(q,\rho)$ represents the controller transfer function. Here, the controller is parametrized by a parameter vector, namely $\rho \in \mathbb{R}^n$ such as $\rho = [\rho_1\ \rho_2\ \ldots\ \rho_n]$, and it belongs to a predefined and fixed controller class, defined as

$$\mathcal{C} \triangleq \left\{C(q,\rho) : \rho \in \mathcal{P} \subseteq \mathbb{R}^n\right\}. \tag{4}$$

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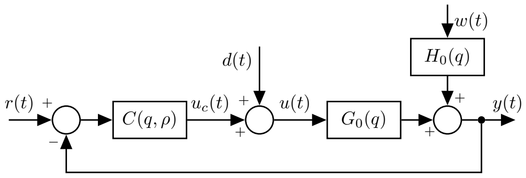

Fig. 1. Closed-loop system's block diagram.

The closed-loop system is presented in Fig. 1. Both reference and disturbance signals are considered to be quasi-stationary processes [12] uncorrelated with the output noise:

$$\bar{E}\big[r(t)w(t)\big] = 0, \qquad \bar{E}\big[d(t)w(t)\big] = 0, \tag{5}$$

where the operator $\bar{E}[\cdot]$ is defined as [12]:

$$\bar{E}[x(t)] \triangleq \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E[x(t)]. \tag{6}$$

Under the closed-loop setup shown in Fig. 1, the system's output can be expressed by the following equation:

$$y(t,\rho) = T(q,\rho)r(t) + Q(q,\rho)d(t) + S(q,\rho)H_0(q)w(t), \tag{7}$$

with

$$T(q,\rho) \triangleq \frac{C(q,\rho)G_0(q)}{1 + C(q,\rho)G_0(q)}, \tag{8}$$

$$Q(q,\rho) \triangleq \frac{G_0(q)}{1 + C(q,\rho)G_0(q)}, \tag{9}$$

$$S(q,\rho) \triangleq \frac{1}{1 + C(q,\rho)G_0(q)}. \tag{10}$$

The problem addressed on this work is to tune the controller in order to optimize the performance on load disturbance rejection trough a data-driven approach, i.e. using only batches of input and output data, without the need of a mathematical process' model.

The performance is specified using a *reference model* $Q_d(q)$ which describes the desired output to some disturbance signal. Then, an optimization problem is formulated to minimize the difference between $Q(q,\rho)$ and $Q_d(q)$:

$$\rho_{md} = \arg\min_{\rho} J_{md}(\rho), \tag{11}$$

$$J_{md}(\rho) \triangleq \bar{E}\big[Q_d(q)d(t) - Q(q,\rho)d(t)\big]^2. \tag{12}$$

The *ideal controller*, i.e., the one that leads the closed-loop system to behave *exactly* as defined by the reference model $Q_d(q)$, is given by [2], [1]:

$$C_d(q) \triangleq \frac{G_0(q) - Q_d(q)}{G_0(q)Q_d(q)}, \tag{13}$$

$C_d(q) \in \mathcal{C} \rightarrow \exists\rho_d : C(q,\rho_d) = C_d(q)$.

When the ideal controller does not belong to the controller class $\mathcal{C}$, the controller $C(q,\rho_{md})$ is different than $C_d(q)$, but it is the best controller that can be achieved to reach, as close as it can, the desired load disturbance performance [1].

III. Appropriate choice of the reference model

Since $C_d(q)$ depends on $G_0(q)$ and $Q_d(q)$, the latter should be chosen wisely by the user in order to result in a well-formulated problem, where $C_d(q)$ is stable and causal. Indeed, the choice of the reference model $Q_d(q)$ was already addressed in the literature, where [2], [3] proposed a model-based solution and [8] presented a solution for the data-driven scenario, introducing some guidelines for the design of $Q_d(q)$ using a flexible structure. Observe that

$$C_d(q) = \frac{dQ_d(q)nG_0(q) - nQ_d(q)dG_0(q)}{nG_0(q)nQ_d(q)}, \tag{14}$$

where $nF(q)$ denotes the numerator of the transfer function $F(q)$ and $dF(q)$ its denominator. This expression is useful to choose the reference model since it describes how $Q_d(q)$ affects the stability and causality of the ideal controller $C_d(q)$. The following Theorems, based on (14), describe the stability and causality of the ideal controller $C_d(q)$.

*Theorem 1 (**Causality of** $C_d(q)$):* The controller $C_d(q)$ will be proper if and only if [2]:

$$Dg\big[dQ_d(q)nG_0(q) - dG_0(q)nQ_d(q)\big] \leq Dg\big[nQ_d(q)nG_0(q)\big], \quad \Gamma[G_0(q)] = \Gamma\big[Q_d(q)\big]$$

where $Dg[F(q)]$ denotes the degree of the polynomial and $\Gamma[F(q)]$ denotes the relative degree of $F(q)$.

*Theorem 2 (**Stability of** $C_d(q)$):* The controller $C_d(q)$ will be stable if and only if there exists $f(q)$ and $g(q)$ such that [2]:

- $nQ_d(q) = f(q)nG_0^+(q)$;
- $dQ_d(q)nG^-(q) = dG_0(q)f(q) + g(q)nG_0^+(q)$,

with

$$Dg\big[f(q)\big] = Dg\big[dQ_d(q)\big] + Dg\big[nG_0^-(q)\big] - Dg\big[dG_0(q)\big]$$
$$Dg\big[g(q)\big] = Dg\big[dQ_d(q)\big] + 2Dg\big[nG_0^-(q)\big] - Dg\big[dG_0(q)\big]$$

where $nG_0^-(q)$ and $nG_0^+(q)$ represent the minimum phase and non-minimum phase factors of $nG_0(q)$, respectively.

From Theorems 1 and 2 it is clear that the choice of the numerator of $Q_d(q)$ is critical to ensure the causality and stability of $C_d(q)$, and it is highly dependent on $G_0(q)$, which is unknown in a data-driven control design. Then, to accomplish such conditions without using a process model, the alternative proposed in [8] is to use a flexible structure for the reference model as $Q_d(q,\eta) = \bar{F}(q)^T\eta$, where $\eta = \begin{bmatrix}\eta_1 & \eta_2 & \dots & \eta_m\end{bmatrix}^T \in \mathbb{R}^m$ and $\bar{F}(q)$ is an m-vector of rational transfer functions, which also belongs to a predefined and fixed class:

$$\mathcal{Q} \triangleq \big\{Q(q,\eta) : \eta \in \mathcal{M} \subseteq \mathbb{R}^m\big\}. \tag{15}$$

With this choice, a partition of $Q(q,\eta)$ is let free to be identified together with the controller parameters.

Finally, the choice of the data-driven structure for $Q(q,\eta)$ can be accomplished by the user from the guidelines provided in [8] as follows:

- Observe the number of non-minimum phase zeros in $G_0(q)$, denoted by $Dg\big[nG_0^+(q)\big]$;
- Observe the relative degree of $G_0(q)$, i.e., $\Gamma\big[G_0(q)\big]$;

- Choose l fixed zeros to include in $nQ_d(q,\eta)$. Ex: to reject step disturbance signals, a zero should be fixed at 1, and so, $l = 1$;
- Determine the order of $Q_d(q,\eta)$:

$$Dg\big[dQ_d(q,\eta)\big] = 2\big\{\Gamma\big[G_0(q)\big] + Dg\big[nG_0^+(q)\big]\big\} + l - 1; \tag{16}$$

- Design the poles in $\bar{F}(q)$ according to the desired dynamics, e.g. desired settling time;
- Determine the number of free parameters in $nQ_d(q,\eta)$:

$$m = Dg\big[dQ_d(q,\eta)\big] - l - \Gamma\big[G_0(q)\big] + 1. \tag{17}$$

These guidelines are based in some trivial quantities from $G_0(q)$, such as its relative degree and quantity of non-minimum phase zeros. Due to the data-driven approach, this knowledge of the process is not directly available from a model. However, all those quantities can be reached from an analysis of the collected data like the evaluation of the process' delay and the presence of an inverse response. Thus, the appropriate choice of the reference model also remains on a data-driven approach.

IV. The VDFT flexible solution

VDFT is a data-driven control method inspired on the virtual reference approach [13] to optimize the load disturbance rejection [1]. In order to identify both the controller and the reference model, the VDFT solution is rewritten in [8] as the minimization of:

$$J_{vd}(\rho,\eta) = \bar{E}\big\{K(q)\big[Q_d(q,\eta)\big(u(t) + C(q,\rho)y(t)\big) - y(t)\big]\big\}^2, \tag{18}$$

where $K(q)$ is an additional filter used to approximate both $J_{vd}(\rho,\eta)$ and $J_{md}(\rho,\eta)$, where $J_{md}(\rho,\eta)$ is equivalent to (12), but considering a flexible reference model $Q_d(q,\eta)$ [8]. If the controller has a linear parametrization as $C(q,\rho) = \bar{\beta}(q)^T\rho$, with $\bar{\beta}(q)$ being an n-vector of rational transfer functions, then the solution of the optimization problem described in (18) is given iteratively for ρ and η for each iteration i, starting with an initial controller $C(q,\rho_0)$ [8], with the sequential least squares algorithm:

$$\hat{\eta}_{ls}^i = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi_q(t)\varphi_q(t)^T\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\varphi_q(t)y(t)^T\right] \tag{19}$$

$$\hat{\rho}_{ls}^i = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi_c(t)\varphi_c(t)^T\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\varphi_c(t)z(t)^T\right], \tag{20}$$

where

$$\varphi_q(t) = K(q)\bar{F}(q)\big(u(t) + \bar{\beta}(q)^T\rho_{ls}^{i-1}y(t)\big), \tag{21}$$

$$\varphi_c(t) = K(q)\bar{\beta}(q)\big(\bar{F}(q)^T\eta_{ls}^i y(t)\big), \tag{22}$$

$$z(t) = K(q)\big[y(t) - \bar{F}(q)^T\eta_{ls}^i u(t)\big], \tag{23}$$

that is guaranteed to converge to a local minimum at least [14], [12]. The formulation of VDFT is based on noise-free signals. When signals are corrupted with noise, the estimates are not consistent and an instrumental variable should be used to cope with this problem. However, it is well known that this approach results in estimates with large variance. In the reference tracking problem, the OCI method stands as an alternative to the VRFT approach to improve these properties through a prediction error approach.

Inspired by the OCI solution for reference tracking, this work proposes a new approach for the load disturbance problem. Based on the prediction error identification, the OCI-D stands as a data-driven method that aims to improve the statistical properties of the data-driven controller design for load disturbance rejection when compared to the VDFT.

V. Prediction error control design for load disturbance rejection

The OCI method can be seen as the identification of the ideal controller through the prediction error approach [11]. The idea is to rewrite system's equation (1) substituting $G_0(q)$ by its relation with the ideal controller and the reference model. Since the reference model is usually fixed, when the identification is performed, a parameterized controller is identified. The same reasoning is applied here for disturbance rejection, with the difference that part of the reference model has also to be identified.

Consider we want to identify the ideal controller for load disturbance rejection. From (13) we can isolate $G_0(q)$ as

$$G_0(q) = \frac{Q_d(q)}{1 - C_d(q)Q_d(q)}. \tag{24}$$

Rewriting (1), a model can be obtained as a function of the controller and the reference model parameters:

$$y(t,\xi) = \frac{Q_d(q,\eta)}{1 - C(q,\rho)Q_d(q,\eta)}u(t) + H(q,\theta)w(t), \tag{25}$$

with $\xi = \begin{bmatrix}\rho^T & \eta^T & \theta^T\end{bmatrix}^T$, where $\rho \in \mathbb{R}^n$ are the controller parameters, $\eta \in \mathbb{R}^m$ are the reference model parameters and $\theta \in \mathbb{R}^t$ are additional noise model parameters.

Using a batch of data collected from either an open or closed-loop experiment, with $Z^N = [u(1)\ \ y(1)\ \ \ldots\ \ u(N)\ \ y(N)]$, it is possible to estimate ξ by minimizing the prediction error loss function:

$$\hat{\xi} = \arg\min_{\xi} J_{ocid}(\xi),$$

$$J_{ocid}(\xi) = \sum_{t=1}^{N}\big(y(t) - \hat{y}(t,\xi)\big)^2, \tag{26}$$

where $\hat{y}(t,\xi)$ is the optimal one-step-ahead predictor [12]:

$$\hat{y}(t,\xi) = \frac{1}{H(q,\theta)}\frac{Q_d(q,\eta)}{1 - C_d(q,\rho)Q_d(q,\eta)}u(t) + \big[1 - H(q,\theta)^{-1}\big]y(t). \tag{27}$$

When the reference model's and the controller's classes ($\mathcal{Q}$ and $\mathcal{C}$) are chosen so the Model Reference Matching is possible, it can be said that the following assumption holds:

*Assumption 1 (**Flexible Model Matching Condition**):* There exists a pair (ρ_d, η_d) such that:

$$\exists\ \rho_d, \eta_d : C(q, \rho_d) = \frac{G_0(q) - Q_d(q, \eta_d)}{G_0(q) Q_d(q, \eta_d)}. \qquad (28)$$

If Assumption 1 is satisfied, the problem of the identification of $C(q, \rho)$ and $Q_d(q, \eta)$ by minimizing $J_{ocid}(\xi)$ results in the same problem as the minimization of $J_{md}(\rho, \eta)$.

The main difference of this work, compared to the original OCI method for reference tracking, is that the complexity of the optimization problem is considerably higher, due to the dependence of the predictor on the controller's and reference model's parameters, which does not allow the use of standard structures and identification softwares. Thus, a specific algorithm is developed to identify ρ and η.

This algorithm is sliced into two optimization steps. The first one is a gradient descent method with starting point obtained from the parameters identified by VDFT. Through that, new parameters are estimated iteratively by

$$\hat{\xi}_{i+1} = \hat{\xi}_i - \gamma \nabla J_{ocid}(\hat{\xi}_i), \qquad (29)$$

where γ is a dynamic step that increases or decreases 1% of its value on each round of the algorithm if the cost function decreases or increases, respectively, and $\nabla J_{ocid}(\hat{\xi}_i)$ is the cost function gradient. The second optimization step is a Newton's method that goes on from the parameters estimated in the gradient descent method. In this case, new parameters are estimated iteratively by

$$\hat{\xi}_{i+1} = \hat{\xi}_i - H^{-1}_{J_{ocid}}(\hat{\xi}_i) \nabla J_{ocid}(\hat{\xi}_i), \qquad (30)$$

where $H_{J_{ocid}}(\hat{\xi}_i)$ is an approximation of the Hessian matrix given by [12]:

$$H_{J_{ocid}}(\hat{\xi}_i) = \frac{1}{N} \sum_{t=1}^{N} \nabla J_{ocid}(t, \hat{\xi}_i) \nabla J^T_{ocid}(t, \hat{\xi}_i). \qquad (31)$$

Despite the computational complexity to solve the optimization problem, the main advantage of the OCI-D method is that all the theoretical and favorable properties of prediction error applies for the identification of $C(q, \rho)$ and $Q_d(q, \eta)$. Therefore, it is possible to obtain *unbiased* estimates with lower variance errors when Assumption 1 is satisfied [14], [12], if compared to least-squares or instrumental variable solutions, as the ones used by VDFT.

VI. Numerical Examples

To exploit the statistical properties of OCI-D, two numerical examples are performed comparing the results achieved by VDFT and the instrumental variable approach against the prediction error strategy when the batch of data is affected by noise. The performances are rated through two examples, being the first one the case when the Flexible Matching Condition is satisfied and the second one the case when it does not.

A. Matched case

Consider an LTI process that can be described as a first order transfer function running in closed loop with a proportional-integral controller:

$$G(q) = \frac{0.5}{q - 0.9} \qquad C(q) = \frac{0.2q - 0.14}{q - 1}. \qquad (32)$$

A reference model for disturbance rejection is specified following the guidelines presented in Section III. Since the plant is minimum-phase, $Deg[nG^+(q)] = 0$. Being $\Gamma[G(q)] = 1$ and addressing the controller design to reject a step disturbance, which implies in $l = 1$, the denominator order of $Q_d(q, \eta)$ given by (16) is set to $Deg[dQ_d(q, \eta)] = 2$. Finally, (17) sets that the number of free parameters in the numerator of the reference model should be $m = 1$.

Setting the poles' position as $p_i = 0.8$ to achieve a faster disturbance rejection compared to the system response and tuning a PI controller, the structures of the reference model and the controller are given by:

$$Q_d(q, \eta) = \frac{\eta_0(q - 1)}{(q - 0.8)^2} \qquad C(q, \rho) = \frac{\rho_0 q + \rho_1}{q - 1} \qquad (33)$$

For this case, the ideal controller specified by (13) using the reference model (33) results in a transfer function like:

$$C(q, \rho_d) = \frac{a_0 q^2 + a_1 q + a_2}{q - 1}, \qquad (34)$$

where $a_0 = (0.5 - \eta_0)$. Hence, $\eta_0 = 0.5$ must hold to guarantee that the ideal controller is causal and belongs to the predefined PI controller class.

With this set, a Monte Carlo experiment of 1000 iterations is evaluated to identify the parameters of the controller and the reference model, looking into the performances of VDFT, VDFT with instrumental variables (VDFT-IV) and OCI-D. On each round, a batch of data is collected from the closed loop noisy system with a signal-to-noise ratio of 7dB when a step load disturbance is applied at the process input to controller design through VDFT and OCI-D. The instrumental variable for VDFT-IV is obtained from another experiment, equal to the first one. Each batch of data with 100 samples is then used by VDFT to estimate the initial parameters, being the average results as:

$$C_{vdft}(q, \hat{\rho}) = \frac{0.39q - 0.32}{q - 1} \qquad Q_{d_{vdft}}(q, \hat{\eta}) = \frac{0.65(q - 1)}{(q - 0.8)^2}$$

Those parameters are a good starting point for OCI-D, and alongside with the batch of data it allows to run the optimization method. After running OCI-D, a new set of parameters is estimated for the controller and the reference model, being the average results equal to:

$$C_{ocid}(q, \hat{\rho}) = \frac{0.60q - 0.52}{q - 1} \qquad Q_d(q, \hat{\eta}) = \frac{0.51(q - 1)}{(q - 0.8)^2}$$

The step response of every closed-loop system identified on this simulation is presented in Fig. 2, where the performances of VDFT, VDFT-IV and OCI-D are compared to the noiseless and to the initial responses. It is visible that

VDFT is biased, since the mean controller and reference model pair is not the ideal values. The IV solution adopted with VDFT is unbiased, but the variance is high and finally OCI-D results are unbiased and show lower variance. The noiseless response presented in graphs is also the response of (33) with $\eta_0 = 0.5$.

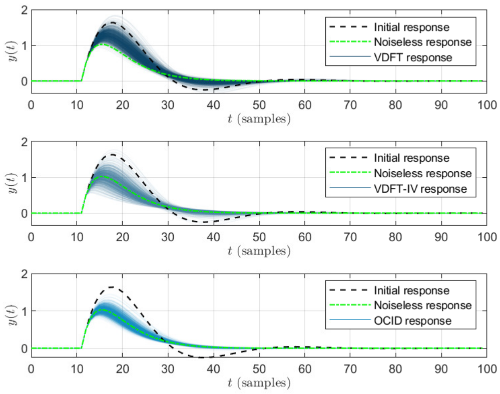

Fig. 2. Step responses of the Monte Carlo simulation with VDFT, VDFT-IV and OCI-D for the matched case.

A further analysis is performed from the box-plots presented in Fig. 3, where the statistics of the three methods are set side by side in an evaluation of the error to the noise free response, which can be written as a cost function like

$$J_n(\rho, \eta) = \sum \left(y_n(t) - \hat{y}(t, \rho, \eta)\right)^2, \tag{35}$$

being $y_n(t)$ the closed loop response achieved with a noiseless batch of data. Again, it is noticeable that OCI-D achieves the best results, showing lower median and variance than VDFT and VDFT-IV.

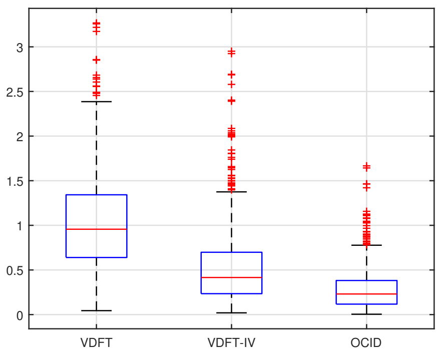

Fig. 3. Error to noise free response box plot comparison between VDFT, VDFT-IV and OCI-D for the matched case.

This is also supported by Table I, where the mean ($\overline{x}$) of the identified parameters are shown alongside with the standard deviation (σ) values inside parenthesis. It is important to notice that all methods have the same noise free output, since in this case the estimated parameters match the condition of the ideal controller. Being $Tr(.)$ the trace operator, an evaluation of the covariance matrix of the parameters vector results in Tr(VDFT)=0.02, Tr(VDFT-IV)=0.09 and Tr(OCI-D)=0.01. Through all those perspectives, OCI-D stands with the best results, endorsing the expected behavior due to the prediction error approach.

TABLE I

PARAMETERS VECTORS STATISTICS $\overline{x}(\sigma)$ FOR THE MATCHED CASE.

	Noise free	**VDFT**	**VDFT-IV**	**OCI-D**
ρ_0	0.60	0.39 (0.09)	0.63 (0.20)	0.60 (0.09)
ρ_1	-0.52	-0.33 (0.08)	-0.54 (0.19)	-0.52 (0.08)
η_0	0.50	0.65 (0.09)	0.51 (0.10)	0.51 (0.05)

B. *Mismatched case*

Consider now an LTI process described as a second order transfer function running in closed loop with a PI controller designed for reference tracking as:

$$G(q) = \frac{0.1}{(q-0.7)(q-0.9)} \qquad C(q) = \frac{0.3q - 0.24}{q-1} \tag{36}$$

Once more, the guidelines presented in Section III are followed to specify the reference model structure. Since there are no non-minimum phase zeros, $Deg[nG^+(q)] = 0$. Being $\Gamma[G(q)] = 2$ and addressing the controller design to reject a step disturbance, or $l = 1$, the denominator order of $Q_d(q, \eta)$ given by (16) is set to $Deg[dQ_d(q, \eta)] = 4$. In its turn, the number of free parameters in the numerator of the reference model given by (17) sets $m = 2$. Finally, setting the poles position as $p_i = 0.6$ to achieve a faster disturbance rejection compared to the system response and tuning again a PI controller, the structures of the reference model and the controller class are given by:

$$Q_d(q, \eta) = \frac{(\eta_0 q + \eta_1)(q-1)}{(q-0.6)^4} \qquad C(q, \rho) = \frac{\rho_0 q + \rho_1}{q-1} \tag{37}$$

For this case, the ideal controller specified by (13) results in a transfer function like:

$$C(q, \rho_d) = \frac{a_0 q^2 + a_1 q + a_2}{q^2 + b_1 q + b_2}, \tag{38}$$

so the ideal controller clearly does not belong to the predefined controller class of the PI controller.

Likewise the previous example, a Monte Carlo experiment of 1000 iterations is evaluated to explore the performances of all three methods. On each round, a batch of data is collected from the closed loop noisy system with a signal-to-noise ratio of 10dB when a step load disturbance is applied at the process input. This batch of data with 100 samples is then used by VDFT to estimate the initial parameters, being the average results as follow:

$$C_{vdft}(q, \hat{\rho}) = \frac{0.63q - 0.56}{q-1} \qquad Q_{d_{vdft}}(q, \hat{\eta}) = \frac{(0.03q + 0.08)(q-1)}{(q-0.6)^2}$$

Using those parameters as starting point alongside with the batch of data to run OCI-D, a new set of parameters is estimated for the controller and the reference model, being the average results equal to:

$$C_{ocid}(q, \hat{\rho}) = \frac{0.59q - 0.52}{q-1} \qquad Q_{d_{ocid}}(q, \hat{\eta}) = \frac{(0.03q + 0.09)(q-1)}{(q-0.6)^2}$$

The step disturbance response of every closed-loop system on this simulation are presented in Fig. 4, where the performances of VDFT, VDFT-IV and OCI-D are compared to the noiseless and to the initial responses. Again, it is possible to notice that the OCI-D results show a closer behavior to the noiseless response and lower variance.

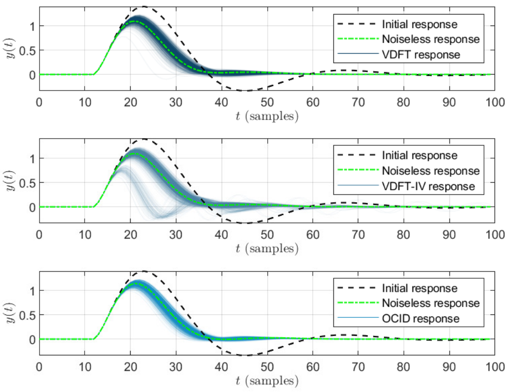

Fig. 4. Step responses of the Monte Carlo simulation with VDFT, VDFT-IV and OCI-D for the mismatched case.

Looking at Fig. 5, where the statistics of the three methods are set side by side in an evaluation of the error to the noise free response defined by (35), we see that OCI-D achieves the best results one more time, showing the lowest median and variance compared to VDFT and VDFT-IV.

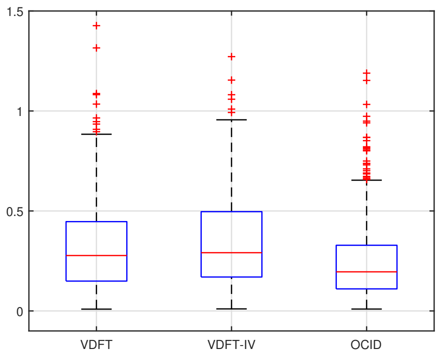

Fig. 5. Error to noise free response box plot comparison between VDFT, VDFT-IV and OCI-D for the mismatched case.

Table II presents the statistics of the identified parameters for the mismatched case, and finally, the evaluation of the covariance matrix of the parameters' vector results in Tr(VDFT)=0.02, Tr(VDFT-IV)=0.13 and Tr(OCI-D)=0.01. Likewise in the matched case, OCI-D stands with the best results and statistical properties.

VII. Concluding Remarks

This work introduced a novel methodology to data-driven control design for load disturbance rejection. The controller identification was formulated in a prediction error approach, using the data-driven solution for the flexible choice of the reference model, and identifying some of its parameters alongside the controller. Also, the paper discussed the complexity of the resulting optimization problem, which is rather more complex than other currently available methodologies.The method's statistical properties were also explored under some specific conditions and numerical examples exhibited the comparison between VDFT, VDFT-IV and OCI-D, showing that the the proposed methodology outperforms both least squares and instrumental variable solutions when data is corrupted with noise.

TABLE II
Parameters vectors statistics $\overline{x}(\sigma)$ for the mismatched case.

	VDFT	**VDFT-IV**	**OCI-D**
ρ_0	0.63 (0.08)	0.71 (0.25)	0.59 (0.05)
ρ_1	-0.56 (0.07)	-0.63 (0.22)	-0.52 (0.04)
η_0	0.03 (0.05)	0.05 (0.07)	0.03 (0.04)
η_1	0.08 (0.06)	0.06 (0.09)	0.09 (0.05)

References

[1] D. Eckhard, L. Campestrini, and E. C. Boeira, "Virtual disturbance feedback tuning," *IFAC Journal of Systems and Control*, vol. 3, pp. 23–29, 2018.

[2] G. Szita and C. Sanathanan, "Model matching controller design for disturbance rejection," *Journal of the Franklin Institute*, vol. 333, no. 5, pp. 747 – 772, 1996.

[3] ——, "A model matching approach for designing decentralized MIMO controllers," *Journal of the Franklin Institute*, vol. 337, no. 6, pp. 641–660, 2000.

[4] D. Chen and D. E. Seborg, "PI/PID controller design based on direct synthesis and disturbance rejection," *Industrial & engineering chemistry research*, vol. 41, no. 19, pp. 4807–4822, 2002.

[5] J.-C. Jeng and G.-P. Ge, "Disturbance-rejection-based tuning of proportional–integral–derivative controllers by exploiting closed-loop plant data," *ISA transactions*, vol. 62, pp. 312–324, 2016.

[6] G. O. Guardabassi and S. M. Savaresi, "Data-based simultaneous design of composite feedback-feedforward controllers: a virtual input direct design approach," in *1997 european control conference (ecc)*. IEEE, 1997, pp. 382–387.

[7] J. D. Rojas, J. A. Baeza, and R. Vilanova, "Three degrees of freedom virtual reference feedback tuning design and its application to wastewater treatment plant control," in *Proceedings of the 18th IFAC World Congress*. IFAC, 2011, pp. 7144–7149.

[8] V. Bordignon and L. Campestrini, "Improving the choice of disturbance reference model in data-driven control methods," in *Proceedings of the XXII Brazilian Conference on Automatic Control*. Sociedade Brasileira de Automática, 2018, pp. 1–7.

[9] ——, "Data-driven PID control tuning for disturbance rejection in a hierarchical control architecture," in *Proceedings of the 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control*. IFAC, 2018, pp. 569–574.

[10] R. W. da Silva and D. Eckhard, "Extension of the correlation-based tuning method for load disturbance rejection," in *Proceedings of the 58th IEEE Conference on Decision and Control (CDC)*. IEEE, 2019, pp. 8386–8391.

[11] L. Campestrini, D. Eckhard, A. S. Bazanella, and M. Gevers, "Data-driven model reference control design by prediction error identification," *Journal of the Franklin Institute*, vol. 354, no. 6, pp. 2628–2647, 2017.

[12] L. Ljung, *System identification: theory for the user*, 2nd ed. Englewoods Cliff: Prentice Hall, 1999.

[13] M. C. Campi, A. Lecchini, and S. M. Savaresi, "Virtual reference feedback tuning: a direct method for the design of feedback controllers," *Automatica*, vol. 38, no. 8, pp. 1337–1346, 2002.

[14] T. Söderström and P. Stoica, *System identification*. Englewoods Cliff: Prentice Hall, 1989.